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A Preconditioned Multigrid Method for Efficient Simulation of Three-dimensional Compressible and Incompressible Flows

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Abstract

To develop an efficient and robust aerodynamic analysis method for numerical optimization designs of wing and complex configuration, a combination of matrix preconditioning and multigrid method is presented and investigated. The time derivatives of three-dimensional Navier-Stokes equations are preconditioned by Choi-Merkle preconditioning matrix that is originally designed for two-dimensional low Mach number viscous flows. An extension to three-dimensional viscous flow is implemented, and a method improving the convergence for transonic flow is proposed. The space discretizaition is performed by employing a finite-volume cell-centered scheme and using a central difference. The time marching is based on an explicit Runge-Kutta scheme proposed by Jameson. An efficient FAS multigrid method is used to accelerate the convergence to steady-state solutions. Viscous flows over ONERA M6 wing and M100 wing are numerically simulated with Mach numbers ranging from 0.010 to 0.839. The inviscid flow over the DLR-F4 wing-body configuration is also calculated to preliminarily examine the performance of the presented method for complex configuration. The computed results are compared with the experimental data and good agreement is achieved. It is shown that the presented method is efficient and robust for both compressible and incompressible flows and is very attractive for aerodynamic optimization designs of wing and complex configuration.

Keywords: Navier-Stokes equations; preconditioning method; multigrid method; numerical simulation

1 Introduction

Over the past two decades, the time-marching algorithms have been widely used for compressible flow simulation solving Euler and Navier-Stokes (N-S) equations. However, it is proved by practice that these "standard" numerical schemes for the compressible equations are not applicable for incompressible flow, and do not converge to the solution of the incompressible equations as the Mach number approaches zero[1].

To overcome this difficulty, several methods were developed for solving nearly incompressible flow problems. Among them, the pseudo-compressibility[2] method and preconditioning method[1,3-6] may be the most popularly used approaches. Due to the assumption of incompressibility, the pseudo-compressibility can only be used for low-speed flow simulation, whereas the preconditioning method is appropriate for both compressible and incompressible flows. The development of the preconditioning method is motivated by two main considerations. First, the actual flow can contain both compressible and incompressible flows simultaneously. Second, it is preferable to develop such a method that is suit for flows at all flow regimes.

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Several inviscid and viscous preconditioning methods were available in the past few years, such as Choi-Merkle's[3], Turkle's[1], Van Leer's[4] and Allmaras's[5] methods, etc. The viscous preconditioning method developed by Choi and Merkle has been proved to be efficient for two-dimensional low Mach number flows. However, for compressible flows, this method seems to have little effect on accelerating the convergence.

In present work, Choi-Merkle preconditioning method is extended to solve three-dimensional flows. An improvement is proposed for accelerating the calculation of three-dimensional transonic flows. Furthermore, the incorporation of matrix preconditioning with an FAS multigrid method is implemented, which results in a very efficient approach for the simulations of both compressible and incompressible flows. The developed method is expected to use as an efficient and robust method for aerodynamic optimization designs of wing and wing-body configuration.

2 Computation Method

2.1 Governing equation

After introducing the matrix preconditioning, the non-dimensional form of the three-dimensional compressible N-S equations can be written as

$$\boldsymbol{P}\frac{\partial \boldsymbol{W}}{\partial t}+\frac{\partial(\boldsymbol{E}-\boldsymbol{E}_{\mathrm{v}})}{\partial x}+\frac{\partial(\boldsymbol{F}-\boldsymbol{F}_{\mathrm{v}})}{\partial y}+\frac{\partial(\boldsymbol{G}-\boldsymbol{G}_{\mathrm{v}})}{\partial z}=\boldsymbol{0} \quad (1)$$

where, $\boldsymbol{P}$ is the preconditioning matrix (or preconditioner) and will take various forms depending on different choices. When $\boldsymbol{P}$ is an identity matrix, Eq.(1) recovers to the standard (non-preconditioned) form. The additional vectors in Eq.(1) are

$$\left.\begin{aligned}
&\boldsymbol{W}=(\rho \;\; \rho u \;\; \rho v \;\; \rho w \;\; \rho E)^{\mathrm{T}}\\
&\boldsymbol{E}=(\rho u \;\; \rho u^2+p \;\; \rho uv \;\; \rho uw \;\; \rho Hu)^{\mathrm{T}}\\
&\boldsymbol{F}=(\rho v \;\; \rho uv \;\; \rho v^2+p \;\; \rho vw \;\; \rho Hv)^{\mathrm{T}}\\
&\boldsymbol{G}=(\rho w \;\; \rho wu \;\; \rho wv \;\; \rho w^2+p \;\; \rho Hw)^{\mathrm{T}}\\
&\boldsymbol{E}_{\mathrm{v}}=(0 \;\; \tau_{xx} \;\; \tau_{xy} \;\; \tau_{xz} \;\; \beta_x)^{\mathrm{T}}\\
&\boldsymbol{F}_{\mathrm{v}}=(0 \;\; \tau_{yx} \;\; \tau_{yy} \;\; \tau_{yz} \;\; \beta_y)^{\mathrm{T}}\\
&\boldsymbol{G}_{\mathrm{v}}=(0 \;\; \tau_{zx} \;\; \tau_{zy} \;\; \tau_{zz} \;\; \beta_z)^{\mathrm{T}}
\end{aligned}\right\} \quad (2)$$

where ρ denotes the density, u, v and w denote the components of velocity vector, and E denotes the total energy per unit mass. Pressure and temperature are given by the equations of state for perfect gas:

$$\left.\begin{aligned}
&p=\rho(\gamma-1)[E-0.5(u^2+v^2+w^2)]\\
&T=p/\rho
\end{aligned}\right\} \quad (3)$$

where γ is the ratio of specific heat and is taken as 1.4 for air. The viscous shear stresses and the heat fluxes are of the form:

$$\left.\begin{aligned}
&\tau_{xx}=2\mu u_x+\lambda(u_x+v_y+w_z)\\
&\tau_{yy}=2\mu v_y+\lambda(u_x+v_y+w_z)\\
&\tau_{zz}=2\mu w_z+\lambda(u_x+v_y+w_z)\\
&\tau_{xy}=\tau_{yx}=\mu(u_y+v_x)\\
&\tau_{yz}=\tau_{zy}=\mu(v_z+w_y)\\
&\tau_{xz}=\tau_{zx}=\mu(u_z+w_x)\\
&\beta_x=u\tau_{xx}+v\tau_{xy}+w\tau_{xz}+kT_x\\
&\beta_y=u\tau_{yx}+v\tau_{yy}+w\tau_{yz}+kT_y\\
&\beta_z=u\tau_{zx}+v\tau_{zy}+w\tau_{zz}+kT_z
\end{aligned}\right\} \quad (4)$$

where k is the coefficient of thermal conductivity and is determined by using the assumption of constant Prandtl number. The bulk viscosity λ is taken to be $-2\mu/3$ according to Stokes's hypothesis. For turbulent flows, the total viscosity μ is calculated as

$$\mu=\mu_{\mathrm{l}}+\mu_{\mathrm{t}} \quad (5)$$

where μ_{l} is the molecular viscosity calculated by Sutherland law, and μ_{t} is the eddy viscosity determined by turbulence model. Then, Eq.(1) can be called preconditioned Reynolds averaged N-S equation. For the aerodynamic design of wing, the attached flows with small separation region are frequently considered. In considerations of simplicity and efficiency, the Baldwin-Lomax algebraic turbulence model is used for all the calculations of the present work.

Rather than using the conservative variables $\boldsymbol{W}=(\rho \;\; \rho u \;\; \rho v \;\; \rho w \;\; \rho E)^{\mathrm{T}}$, the primitive variables $\boldsymbol{Q}=(p \;\; u \;\; v \;\; w \;\; T)^{\mathrm{T}}$ are frequently used for viscous flows, especially for low Mach number flows. Then Eq.(1) can be written as

$$\boldsymbol{\Gamma}\frac{\partial \boldsymbol{Q}}{\partial t}+\frac{\partial(\boldsymbol{E}-\boldsymbol{E}_{\mathrm{v}})}{\partial x}+\frac{\partial(\boldsymbol{F}-\boldsymbol{F}_{\mathrm{v}})}{\partial y}+\frac{\partial(\boldsymbol{G}-\boldsymbol{G}_{\mathrm{v}})}{\partial z}=\boldsymbol{0} \quad (6)$$