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Research paper

Influence of edge additions on the synchronizability of oscillatory power networks

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The influence of edge-adding number and edge-adding distance on synchronization of oscillatory power network is investigated. Here we study how the addition of new links impacts the emergence of synchrony in oscillatory power network, focusing on ring, and tree-like topologies. Numerical simulations show that the impact of distance of adding edges whether homogeneous (generators to generators or consumers to consumers) or heterogeneous (generators to consumer nodes and vice versa) edges is not obvious on the synchronizability of oscillatory power network. However, for the edge-adding number, it is observed that the bigger heterogeneous edge-adding number, the stronger synchronizability of power network will be. Furthermore, the homogeneous edge-adding number does not affect the synchronizability of oscillatory power network.

1. Introduction

The emergence of synchronization in a network of coupled oscillators is a fascinating subject in various scientific disciplines [1,2]. It is commonly agreed that the Kuramoto model and its variations are useful tools to study the dynamics of coupled oscillators. In particular, a coupled oscillatory model with dissimilar natural frequencies can be served as a realistic physical model for a power grid. Furthermore, coupled oscillatory model has interpreted as one of the major model stimulating the study of synchronization phenomena in the field of network dynamics. Studying synchronization of coupled oscillators has proven to be particularly useful in modeling complex systems and uncovering generic mechanisms behind synchronization processes.

For power grid, alternating voltage of the power plants is required to be synchronized around a certain specific frequency, otherwise severe problems like large blackouts are bound to occur in a large area [3]. So far, several authors have investigated synchronization dynamics in power networks. In the survey work [4], it has emphasized that the role of network topology in the synchronization of power systems, they derive the minimum coupling strength to ensure frequency synchronization. Their results reveal that link removal have important consequences to synchronization dynamics in case of cascading failure. [5] has presented a unique concise, and closed-form condition for synchronization of dynamic network, this condition is exact for various interesting network topologies and parameters. Dirk and Marc have studied how the addition of individual links impacts the emergence of synchrony [6]. They showed that adding new links may not only promote but also destroy synchrony. However, a power network is heterogeneous, being comprised of nodes with different

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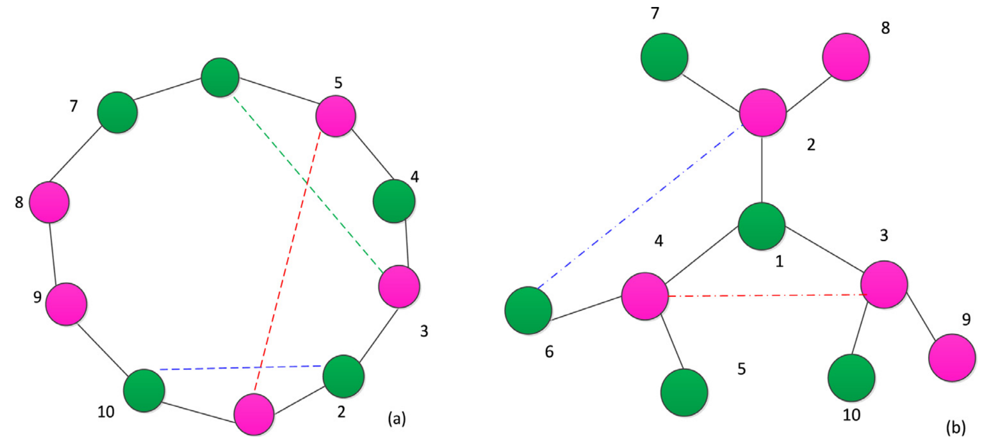

Fig. 1. Adding edges with different distances in networks (a) ring network (b) tree-like network.

physical properties. Especially, generator and consumer nodes have dissimilar frequencies [7–9]. So it is necessary to take into account the properties of edges among different type nodes. On the other hand, the use of renewable and more distributed power sources steadily rises, thus requiring one to add more new transmission lines with different distances in the near future [10]. Note that such lines have to reflect the future demand for transmissions, but at the same time global synchrony has to be ensured [11]. This raises the key question "How do the distance and number of adding new links affect the synchronizability?" In fact, researchers have studied the impact of edge-adding on synchronizability of networks generated from ring networks [12]. The results indicated that the influence of edge addition on synchronizability is neither uniform nor monotone. However, oscillatory power network is a kind of special network with different characteristic edges [13-15]. In the present work, we will continue along this line and analyze the synchronizability of an oscillatory power network with different characteristic edge additions.

Motivated by the above discussion, this paper studies the additions of different characteristic edge in a class of oscillatory power network. Here, we focus on ring and tree-like networks. For an original network, adding new edges on pairs of randomly chosen nodes from among all possible node pairs. Our aim is to explore the influence of both the distance and the number of edge additions on the synchronizability of the oscillatory power network.

2. Couple oscillator model for power grids

For simplicity, we consider an undirected and unweighted oscillatory power model where each node is one of two types of nodes, generators or consumers. Suppose the network with N nodes, labeled as $1, 2, \ldots, N$, according to a counterclockwise or clockwise arrangement.

According to the Ref. [12], we also use the same distance definition. The distance between node i and node j, denoted as d_{ij}, is given by:

$$d_{ij} = \min(j - i, \mathrm{mod}(i + N - j, N)), \; j > i$$

Where, $\mathrm{mod}(m, n)$ denotes modulo function, which returns the remainder after the number m is divided by the divisor n, and the result has the same sign as the divisor. Furthermore, the connection is symmetric, thus $d_{ij} = d_{ji}$. For the examples a ring and a tree-like networks are illustrated in Fig. 1, respectively. Fig. 1.(a) represents distances between node 2 and node 10, node 3 and node 6, node 1 and node 5, given by $d_{2,10} = 2$ $d_{36} = 3$ $d_{15} = 4$, respectively. In Fig. 1.(b), one can get $d_{3,4} = 2, d_{2,6} = 3$

In order to show crucial aspects of the oscillator dynamics of power grids and collective phenomena emerging, we consider coarse-scale oscillator models of power grids [16]. Each node is characterized by the same type equation of motion with a parameter P_j giving the generated ($P_j > 0$) or consumed ($P_j < 0$) power derived from the physics of synchronous generators and consumers.

The state of each node can be determined by its phase angle and phase velocity. During the regular operation, generators as well as consumers within the grid run with the same frequency $\Omega = 2\pi \times 50Hz$. The phase of each element is depicted as following:

$$\theta_j(t) = \Omega t + \phi_j(t) \tag{1}$$

where $\phi_j(t)$ stands for the phase difference to the reference phase Ωt.

The equation of motion for all $\phi_j(t)$ can be derived from the proper energy balance in the generator, the generated or consumed energy P_i^{source} of each single element must equal the energy sum given or taken from the grid plus the accumulated and dissipated energy of this element.