Commun Nonlinear Sci Numer Simulat 41 (2016) 19–31

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Bifurcation techniques for stiffness identification of an impact oscillator

Maolin Liao[a], James Ing[a,*], Joseph Páez Chávez[a,b], Marian Wiercigroch[a]

[a] Centre for Applied Dynamics Research, School of Engineering, University of Aberdeen, Aberdeen AB24 3UE, UK
[b] Facultad de Ciencias Naturales y Matemáticas, Escuela Superior Politécnica del Litoral, P.O. Box 09-01-5863, Guayaquil Ecuador

ARTICLE INFO

Article history:
Received 5 July 2014
Revised 2 November 2015
Accepted 27 November 2015
Available online 13 February 2016

Keywords:
Impact oscillator
Bifurcation analysis
Stiffness identification
Numerical continuation

ABSTRACT

In this paper, a change in stability (bifurcation) of a harmonically excited impact oscillator interacting with an elastic constraint is used to determine the stiffness of constraint. For this purpose, detailed one- and two-parameter bifurcation analyzes of the impacting system are carried out by means of experiments and numerical methods. This study reveals the presence of codimension-one bifurcations of limit cycles, such as grazing, period-doubling and fold bifurcations, as well as a cusp singularity and hysteretic effects. Particularly, the two-parameter continuation of the obtained codimension-one bifurcations (including both period-doubling and fold bifurcations) indicates a strong correlation between the stiffness of the impacted constraint and the frequency at which a certain bifurcation appear. The undertaken approach may prove to be useful for condition monitoring of dynamical systems by identifying mechanical properties through bifurcation analysis. The theoretical predictions for the impact oscillator are verified by a number of experimental observations.

1. Introduction

Engineering systems with intermittent impacts are very common in science and technology. The effects of impacting behavior can be either beneficial or power-wasting, sometimes even harmful. In order to broaden the application range of such systems and prevent potential hazards, serious investigations are warranted. The simplest model of an intermittent impacting system is a harmonically forced impact oscillator interacting with an elastic constraint. Many previous investigations have revealed a rich bifurcation structure for this or similar systems [1–4], including studies where the impact is modeled by a coefficient of restitution law [5,6]. Methods to control the persistence of a local attractor near grazing periodic and quasiperiodic trajectories [7,8] have been proposed, as has a numerical approach for calculating grazing bifurcations [9]. All of these investigations demonstrate the extensive adaptability of this impact oscillator model in the relevant research field of nonlinear dynamics. In this paper, the system is utilized to explore the relationship between the stiffness of an impacted constraint and frequencies at which bifurcations appear. The core target is to establish a set of dynamical detection techniques via analysis of the collected dynamical signal to determine the properties of the impacted object. Applications for this include non-destructive testing and condition monitoring of structures.

* Corresponding author. Tel.: +44 1224273657.
E-mail addresses: m.liao@abdn.ac.uk (M. Liao), j.ing@abdn.ac.uk (J. Ing), jpaez@espol.edu.ec (J. Páez Chávez), m.wiercigroch@abdn.ac.uk (M. Wiercigroch).

http://dx.doi.org/10.1016/j.cnsns.2015.11.027

Resonance Enhanced Drilling (RED) [10] is the main motivation of the present work. The basic idea of RED technology is to apply an adjustable high frequency dynamic stress generated by axial oscillations, in combination with rotary action, in order to enhance the rate of penetration (ROP) by creating resonance conditions between the drill-bit and the drilled formation [11]. However, due to the heterogeneity and anisotropy of formations, parameters of axial oscillation need to be constantly adjusted so that the resonance condition can be maintained. This requires a real-time monitoring of the properties of the drilled formation, which is a key aspect within the RED technology. The relevant topics have already been investigated for a few years [12–14]. In particular, Ajibose et al. [15,16] used a solid mechanics approach to simulate the vibro-impact drilling process. Myers et al. [17] demonstrated the inverse relationship between amplitude of axial vibration of drill-bit and formation porosity. Esmaeili et al. [18] performed experiments with a drill-bit impacting concrete blocks of varying uni-axial compressive strengths. Richard et al. [19] conducted extensive scratch tests for different types of rocks to compare the intrinsic specific energy and their corresponding uni-axial compressive strengths. Contreras et al. [20] fulfilled the real-time detection of stiffness change using a radial basis function augmented observer formulation. Fontan et al. [21] introduced the Particle Swarm Optimization (PSO) algorithm which can inversely compute unknown parameters through comparison of experimental data and simulation of a dynamic model. All their investigations, to some extent, have validated the feasibility of real-time parameter estimation through indirect measurements.

In this paper, the complicated drill-bit formation interaction is simplified to an impact oscillator contacting with an elastic constraint. The varying formation properties are represented by varying the stiffness of the elastic constraint. Subsequently, the effects of varying stiffness on the dynamic responses of the impact oscillator are tested experimentally, and are further detailed by numerical simulation and continuation. A few studies covered this question [22–24], and showed that the impact stiffness can be an important bifurcation parameter. In this paper, the emphasis is on verifying the effects of the constraint stiffness on the bifurcation frequency by constructing two-parameter bifurcation diagrams.

The paper is organized as follows. Section 2 introduces the experimental set-up and the test procedure. It mentions the problem in setting large-scale test stiffness cases due to the limitation of the experiment rig and its solution. The experimental results, combined with the corresponding numerical simulations, are subsequently discussed. In Section 3, numerical continuation using TC-HAT [25] is conducted to further detail the experimental results discussed in Section 2. Specifically, this section introduces the mathematical formulation of the impact oscillator for TC-HAT, and the corresponding definitions of smooth segments for decomposing trajectories. Subsequently, the numerical continuation for both one-parameter (frequency ratio) and two-parameter (stiffness ratio and frequency ratio) are described. Section 4 compares the results of experiments and numerical continuation, demonstrates the reverse process of estimating the stiffness of the impacted constraint, provides an estimation of the errors and explains their underlying reasons. Concluding remarks are provided in Section 5, and the potential with respect to applications of the bifurcation techniques for stiffness identification is further discussed.

2. Experimental studies

The experimental impact oscillator rig detailed in [1,23,26–29] was used for these investigations, since the dynamic characteristics of this rig are well understood. Previous investigations using this rig only considered bifurcations with a single parameter, such as excitation frequency, amplitude or the gap between oscillator and constraint, but did not consider the effects of stiffness variation. Therefore, experiments detailed in this section focus on codimension-one bifurcations with varying constraint stiffness and excitation frequency.

2.1. Experimental set-up

The physical model of the impact oscillator and the corresponding schematic of the experimental rig are shown in Fig. 1(a) and (b), respectively. In the schematic, the pair of parallel leaf springs supporting a mild steel block are regarded as the first spring k_1 in Fig. 1(a); the elastic beam mounted on a separate column is viewed as the second spring k_2 in Fig. 1(a). The static stiffness parameters of these two springs with different lengths were measured under quasi static deformation using an INSTRON material testing machine. A bolt was used to adjust the gap g between the block and the beam. The harmonic excitation was provided by an electro-dynamic shaker. The experimental data were collected by three sensors; among which, an eddy current probe (x_m in Fig. 1(b)) recorded the displacement of the block; and two accelerometers measured the accelerations of the block (a_m in Fig. 1(b)) and the base (a_b in Fig. 1(b)), respectively. All signals were firstly low pass filtered and amplified; then received and converted by the LabView card; finally the converted data were saved by a connected computer for further analysis.

The mathematical model of the impact oscillator (see Appendix A) is similar to low dimensional models derived for RED; therefore this study into the effects of the secondary stiffness identification can act as a preliminary study on formation identification in a drilling application, where the pair of parallel leaf springs and the steel block can be viewed as a vibrating drill-bit, while the impacted beam represents a drilled formation. Through adjusting the length of the beam, the effective formation strength can be continuously varied, which would not have been possible otherwise.