Commun Nonlinear Sci Numer Simulat 41 (2016) 32–47

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Characterization of chaotic attractors under noise: A recurrence network perspective

Rinku Jacob[a], K.P. Harikrishnan[a,*], R. Misra[b], G. Ambika[c]

[a] Department of Physics, The Cochin College, Cochin-682002, India
[b] Inter University Centre for Astronomy and Astrophysics, Pune-411007, India
[c] Indian Institute of Science Education and Research, Pune-411008, India

ARTICLE INFO

Article history:
Received 31 December 2015
Revised 24 March 2016
Accepted 25 April 2016
Available online 26 April 2016

Keywords:
Recurrence network analysis
Effect of noise on chaotic attractor
Nonlinear analysis of black hole Light curves

ABSTRACT

We undertake a detailed numerical investigation to understand how the addition of white and colored noise to a chaotic time series changes the topology and the structure of the underlying attractor reconstructed from the time series. We use the methods and measures of recurrence plot and recurrence network generated from the time series for this analysis. We explicitly show that the addition of noise obscures the property of *recurrence* of trajectory points in the phase space which is the hallmark of every dynamical system. However, the structure of the attractor is found to be robust even upto high noise levels of 50%. An advantage of recurrence network measures over the conventional nonlinear measures is that they can be applied on short and non stationary time series data. By using the results obtained from the above analysis, we go on to analyse the light curves from a dominant black hole system and show that the recurrence network measures are capable of identifying the nature of noise contamination in a time series.

1. Introduction

It is now well known that many time evolutions in Nature are inherently governed by nonlinear dynamical systems. For a proper understanding of these time evolutions, one often resorts to the methods and tools of nonlinear time series analysis [1], since the information regarding the system in most cases is available in the form of a time series. There are two important properties that are the hallmarks of every dynamical system – determinism and recurrence. The former implies that the future behavior of the system can be accurately predicted, given sufficient knowledge of the current state of the system. By the latter property, the trajectory of a dynamical system tends to revisit every region of the phase space over an interval of time [2]. Hence, these two properties are very important in nonlinear analysis of time series data. Of special interest, in the analysis, is the search for deterministic chaos in the time evolution of the system and the presence of an underlying chaotic attractor. Because of this, many quantifiers from chaos theory [3,4] are constantly being employed in the nonlinear analysis of observational data.

A long standing problem in the time series analysis of real data is the presence of noise, both white and colored, that tend to destroy both the above mentioned properties of any dynamical system underlying the time series. Several aspects

* Corresponding author. Fax: +91-22224954.
E-mail addresses: rinku.jacob.vallanat@gmail.com (R. Jacob), kp_hk2002@yahoo.co.in (K.P. Harikrishnan), rmisra@iucaa.in (R. Misra), g.ambika@iiserpune.ac.in (G. Ambika).

http://dx.doi.org/10.1016/j.cnsns.2016.04.028
1007-5704/

of the effect of noise on synthetic as well as real world data and on the quantifying measures of discrimination have been addressed by many authors since the advent of chaos theory some four decades back. For example, the influence of white noise on the logistic attractor [5], the effect of colored noise on chaotic systems [6], method to distinguish chaos from colored noise in an observed time series [7] and the seminal work on the correlation dimension analysis of colored noise data by Osborne and Provenzale [8], to mention a few. The above studies resulted in the development of surrogate methods [9,10] to discriminate chaos from random noise in real world data. More details on the effect of noise on chaotic systems and discriminating measures can be found in several standard books on chaos [1,3,4].

One important aspect in nonlinear time series analysis that has not received enough attention is to understand how the structure of the underlying attractor is affected by additive noise. One reason for this has been the absense of a tool that is directly correlated with the structural changes in a chaotic attractor. In this paper, we use one such tool, namely, the recurrence plot (RP) and combine it with the newly emerged measures of recurrence networks (RN) [11,12] to undertake a detailed numerical analysis to show how the structure of a chaotic attractor is affected by white and colored noise. This result is then used to study the effectiveness of RN measures in identifying the nature of noise contamination in real world data using astrophysical light curves as example.

The basic idea of RN analysis is that the information inherent in a chaotic time series is mapped onto the domain of a complex network using a suitable scheme. One then uses the statistical measures of the complex network to characterize the underlying chaotic attractor in the time series. It should be noted that here we use the terms "underlying attractor" and "reconstructed attractor" to indicate the attractor constructed from the time series using delay coordinates. We can define a Euclidean distance between any two points on the attractor. On the other hand, the RN is defined in an abstract space consisting of a set of nodes connected by some finite number of links or edges between them. Every point on the attractor is mapped on to a node in the RN and the condition that two nodes are connected is decided by certain criterion as discussed below.

There are two aspects of the RN that make them special for the analysis of time series data. Firstly, since the network measures can be derived from a small number of nodes in the network, the method is suitable for the analysis of short, non stationary data [13]. Secondly, the type of RN called the ϵ - RNs (whose details are given in the next section) that we consider in this work, generally preserve the topology of the embedded attractor from the time series [14]. We show this specifically for the standard Lorenz attractor below. Hence, the topological changes in the underlying attractor due to noise addition are also reflected in the corresponding RN. This, in turn, implies that the RN and the related measures can be effectively used to study how the topological structure of a chaotic attractor is affected by increasing levels of noise contamination.

The above properties of the RN have resulted in a number of practical applications ranging from identification of dynamical transitions in model systems and real data [15,16] to classification of cardio vascular time series [17]. However, there is one area where the RN measures have not been tested properly. For example, a systematic analysis of how the RN measures change with the addition of noise and how effective these measures are in the analysis of real data involving noise, are missing. To our knowledge, there are only two studies in this regard, one using the EEG data [18] and another by Thiel et al., [19] using the RP measures. These factors motivate us for the present analysis.

It should be noted that our aim in this analysis is neither to quantify the amount of noise in a given time series nor to distinguish deterministic nonlinear behavior from randomness in a given time series using any quantifying measure from RN. We focus on the following aspects:

(i) How the contamination of white and colored noise affect the topological structure of a low dimensional chaotic attractor?
(ii) How the basic RN measures change with the addition of white and colored noise to a time series data?
(iii) How effective are the RN measures in identifying the nature of noise contamination in real world data?

We introduce an additional measure for this purpose derived from the RN, which we call the *k-spectrum*. The number of nodes connected by a reference node is called its degree, denoted by k. Here we represent the different k values present in the RN in the form of a discrete spectrum. We show that the structural changes in the attractor due to noise addition can be represented in terms of the variations in the k-spectrum. Also, we use the time series from the standard Lorenz attractor as the prototype to study the effect of noise on synthetic data. The analysis is done by adding different amounts of white and colored noise to the Lorenz data. The results obtained from this is used for the analysis of real world data.

Our paper is organised as follows: In the next section, we discuss the details regarding all the numerical tools used in this analysis, namely, the RP and the RN and the associated measures. In Section 3, we use the time series from the standard Lorenz attractor as an example to construct the RN and study how the network measures change by the addition of different amounts of white and colored noise. The Section 4 is devoted to the analysis of a few light curves from a standard black hole system GRS 1915+105 which are expected to contain different levels of white and colored noise. Conclusions are given in Section 5.

2. Numerical tools: recurrence plot and recurrence network

In this section, we provide details of all the quantifying measures used for this study. RP is a visualisation tool introduced by Eckmann et al., [2]. It is a two dimensional graphical representation of the trajectory of the dynamical system in the form