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Rogue wave formation under the action of quasi-stationary pressure

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ABSTRACT

The process of rogue wave formation on deep water is considered. A wave of extreme amplitude is born against the background of uniform waves (Gerstner waves) under the action of external pressure on free surface. The pressure distribution has a form of a quasi-stationary "pit". The fluid motion is supposed to be a vortex one and is described by an exact solution of equations of 2D hydrodynamics for an ideal fluid in Lagrangian coordinates. Liquid particles are moving around circumferences of different radii in the absence of drift flow. Values of amplitude and wave steepness optimal for rogue wave formation are found numerically. The influence of vorticity distribution and pressure drop on parameters of the fluid is investigated.
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1. Introduction

Rogue waves also referred to as freak waves are waves of large amplitude that arise on sea surface all of a sudden and disappear just as quickly. Their characteristic feature is amplitude criterion according to which the height of rogue waves is twice or more the average height of the surrounding waves [1–4]. Being first considered for ocean waves, the concept has shifted to other fields of physics, such as nonlinear optics [5–8], physics of plasma [9], superfluid helium [10], and Bose-condensate systems [11]. Currently, of great interest is the elucidation of possible mechanisms of rogue wave formation and scenarios of their arising in different physical conditions that ultimately determine parameters and properties of extreme waves.

Rogue wave formation is a nonlinear effect [12] that was studied in the weakly nonlinear approximation within the framework of the nonlinear Schrödinger equation [13–21] and the Dysthe equation [22]. It was found that anomalous amplitude waves may arise as a result of modulation instability of initial perturbations of a definite class (see the reviews [1,4,23]). Dyachenko and Zakharov suggested that focusing of oceanic waves creates only preconditions for rogue wave formation, which is a strongly nonlinear effect. By solving a full system of equations of hydrodynamics they demonstrated that a rogue wave may be formed from a weakly nonlinear Stokes wave [24].

All the theoretical studies mentioned above were carried out for potential wave motion and constant pressure on free liquid surface. These assumptions are justified in the absence of wind. However, rogue waves frequently originate, when the wind impact cannot be neglected. Firstly, the wind changes pressure on the fluid surface, and secondly the wave motion becomes a vortex one. The first factor was taken into consideration in the works [25–30], where the dynamics of weakly nonlinear, narrow bandwidth trains of potential surface waves in the field of variable external pressure defined by the linear theory of wind wave excitation was investigated. Following Miles [31], but within the framework of modulated wave trains, those authors assumed

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the atmospheric pressure at the interface due to wind and the water wave slope to be in phase, which is the necessary condition of energy transport from wind to moving waves. The evolution of the wave train envelope in this case is described by the nonlinear Schrödinger equation with an additional term proportional to the amplitude and ensuring its growth. Consequently, in this case the formation of extreme amplitude waves is determined by both, the modulation instability mechanism [32] and wind.

Yan and Ma studied rogue wave formation within the framework of a full system of equations of hydrodynamics and proposed a phenomenological formula for air flow pressure distribution on free surface [33], where pressure is a linear combination of wave slope and free surface elevation. The authors of [33] also showed that vortex air motion may be neglected in calculations of a maximum-height wave, but they did not analyze the role of liquid vorticity in its formation.

A vortex model of freak wave formation against the background of uniform waves was proposed in the paper [34] based on exact analytical solution of equations of 2D hydrodynamics of an ideal incompressible fluid [35,36]. A unique feature of flows of this class is the dependence of liquid particle motion coordinates on two complex functions that may be arbitrary to a great extent. As a consequence, the model may be used for analysis of different representations of surface pressure as well as liquid vorticity, i.e., taking into account simultaneously both these factors of air flow impact on the surface wave.

A partial exact solution for which the pressure of the free surface varied out of phase with the wave profile was investigated in [34]. Phillips in his book [37], however, emphasized that the phase difference between fluctuations of surface pressure and wave profile in natural environment may be very diverse, and statistical sampling of wave observations does not give unambiguous preference to any value or range of values. In this sense, it should be noted that Yan and Ma restricted applicability of their empirical formula and did not pretend it to be universal.

In the present work we consider the situation with surface pressure distribution qualitatively different from that in [34] within the framework of the class of exact solutions [35,36]. This distribution has a negative pressure pit, so that the elevation of the wave profile is first in phase with the pressure and then in antiphase. In this fashion we simulate a qualitatively self-consistent behavior of wave profile and pressure on it typical for oceanic and laboratory conditions [33,37]. The form of the exact solution is chosen so that pressure should be time independent in Lagrangian variables. In Euler variables, pressure is a function of time but the pit does not change its shape qualitatively. Unlike the nonstationary model considered in [34], the presented model may be called quasi-stationary. The properties of liquid flows for this model are determined by a single complex function.

Modulation instability is regarded to be one of the most probable mechanisms of extreme wave formation. In this respect, it is interesting to note that the role of modulation instability may increase significantly in the crossing sea states, when there are two wave systems (see [16,18,21] and references therein). However, the mechanism of anomalous wave formation considered in our paper is essentially different. It is based on nonuniform pressure distribution over liquid surface. Thus, the pressure gradient plays the part of external force in our model.

A new scenario of rogue wave formation has been found by means of numerical simulations. In the work [34], an extreme wave starts to grow from the Gerstner wave maximum, whereas in the new model it is born in the trough. The initial stage of uniform wave instability is characterized by an increase inside the trough of two local maxima corresponding to the edges of the pressure pit. We have studied the evolution of the vorticity field in the course of rogue wave formation and the influence of pressure drop on its height. Relations for parameters of maximum amplitude wave have been derived. The nonstationary and quasi-stationary models have been compared.

2. Ptolemaic waves on deep water

The equations of 2D hydrodynamics for waves on the surface of an incompressible inviscid fluid in Lagrangian coordinates are written in the following form [38]:

$$\frac{D(X,Y)}{D(a,b)} = \frac{D(X_0,Y_0)}{D(a,b)}, \tag{1}$$

$$X_{tt}X_a + Y_{tt}Y_a = -\frac{1}{\rho}p_a - gY_a, \tag{2}$$

$$X_{tt}X_b + Y_{tt}Y_b = -\frac{1}{\rho}p_b - gY_b, \tag{3}$$

where X, Y are Cartesian coordinates and a, b are Lagrangian coordinates of fluid particles, t is time, ρ is fluid density, p is pressure, g is acceleration of gravity, the subscripts mean differentiation by the corresponding variable, and the subscript "zero" means the value at time $t = 0$.

Eq. (1) is a volume conservation equation and Eqs. (2) and (3) are flow equations. Using the cross differentiation it is possible to exclude the pressure and to obtain the condition of vorticity conservation along the trajectory [38]:

$$(X_{ta}X_b + Y_{ta}Y_b - X_{tb}X_a - Y_{tb}Y_a)_t = 0. \tag{4}$$

Abrashkin and Yakubovich proposed to introduce complex Cartesian coordinates $W = X + iY$ ($\bar{W} = X - iY$) and complex Lagrangian coordinates $\chi = a + ib$ ($\bar{\chi} = a - ib$). Then Eqs. (1) and (4) are equivalent to the conditions of conservation of two

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