Commun Nonlinear Sci Numer Simulat 34 (2016) 86–122

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

# Onto resolving spurious wave reflection problem with changing nonlocality among various length scales

R. Rahman*, J.T. Foster

*Department of Petroleum and Geosystems Engineering, The University of Texas at Austin, TX 78705, United States*

ARTICLE INFO

*Article history:*
Received 14 July 2015
Revised 14 September 2015
Accepted 6 October 2015
Available online 23 October 2015

*Keywords:*
Nonlocal model
Peridynamics
Fractional calculus
Power law
Spurious wave reflection

ABSTRACT

In this work an effective method was proposed in order to resolve the wave reflection problems between local/nonlocal models as well as multiple nonlocal models with varying nonlocality. *Spurious* wave reflection has been a primary concern in developing a robust multiscale–multiresolution model. In the current work a power-law based nonlocal peridynamic model has been proposed in order to mitigate this issue in a versatile manner. The fractional power-law eliminates the spurious wave reflection at the interfaces between local/nonlocal regions or regions with different nonlocalities. By controlling the exponent of the power-law it is possible to vary the frequency components of short or long waves without requiring a large handshake region. Using this underlying idea, $\frac{1}{|x-x'|^{1+\alpha}}$, $\forall\, 0 < \alpha < 2$ can be used as a kernel function in order to define nonlocal interaction between $x$ and $x'$. It was shown that by controlling $\alpha$ it is possible to change the nature of nonlocal interaction within any given cutoff range. Besides power law, Gaussian kernel is another good choice in minimizing the wave reflection issue. However, Gaussian function has some limitations with large variation in nonlocality or waves with higher frequency. In that context the proposed model demonstrated its effectiveness by removing any spurious wave reflections originated in various cases.



## 1. Introduction

Nonlocal continuum theory has become one of the effective ways to solve continuum mechanics problems for various applications including damage and fracture mechanics. There have been different kinds of nonlocal theories developed and used in various applications. In the pioneering nonlocal theory proposed by Kröner, an integral operator was incorporated in the classical elasticity equation [1]. Later on, Eringen proposed nonlocal theory through defining the stress tensor in terms of an integral operator [2]. Considering the limitations (i.e., linearity, introducing failure etc.,) a more generalized nonlocal theory was proposed by Silling which is known as "Peridynamics" [3]. This nonlocal model can handle both linearity and nonlinearity in the materials. The linearized bond-based peridynamic theory is analogous to Kunin's nonlocal model [4,5]. It was also shown that perydinamics can be approximated to weakly nonlocal theory, such as: strain gradient theory [6]. In the current work peridynamics formalism was used to define the nonlocal media. The primary concern in this work is related to the issues with wave reflection in materials. Materials used in real engineering applications are widely affected by their microstructures. Even in the absence of macroscale cracks, the micro-craks, dislocation, grain boundary defects, interfacial defects etc., directly influence the mechanical response of a material. These parameters lead to heterogeneity in a material which causes variation in the nonlocal property. Heterogeneity

* Corresponding author. Tel.: +20 52390559.
*E-mail address:* rezwanrehman@gmail.com (R. Rahman).

http://dx.doi.org/10.1016/j.cnsns.2015.10.003
1007-5704/

might change at the same length scale or at different length scales. Either way the long range property can be different. In order to capture the variation in length scales or change in microstructures etc., nonlocal continuum theory is comparatively more effective than classical elasticity theory. In last few years peridynamics has been extensively used for developing different multiscale modeling frameworks [7–11]. Since the nonlocal models involve interaction among material points residing within a cutoff distance, the characteristics of any expected long range interaction can be imposed by choosing appropriate cutoff distance, i.e., *peridynamic horizon*: $l$ along with the kernel function. Peridynamics can be used at different length scales while varying nonlocality through a hierarchical set of peridynamic horizons. Transition from strong to weaker long range interaction regions or vice versa changes the wave dispersion properties. It is known that nonlocality or length scale are very influential parameters for characterizing wave dispersion in materials [12]. While modeling nonlocal media, lack of smooth transition among long range interactions of different qualities may introduce *spurious* wave reflection at the interface. In order to avoid this issue, the multiscale modeling scheme could be either sequential-hierarchical [7–9] or alternatively, the nonlocal cutoff distance can be varied as smoothly as possible in a concurrent model [13]. The concurrent models are important in the context of varying nonlocality or coupling local/nonlocal regions at a single or multiple length scales. Considering $l(x)$: a function of spatial positions $x$, it was seen that $l(x)$ needs to be varied linearly with $x$ in order to avoid the wave reflection [13]. This means that for wide range of variation in nonlocality the transition has to be slow. This requires an optimum handshake region between different regions with varying nonlocality. In the current work our goal is to enhance this method by incorporating power law based nonlocal kernel functions. It will be shown that by incorporating power law based influence functions with non-integer exponent the nonlocality can be changed very quickly over a small handshake region. The condition for $l(x)$ to be continuous linear function can be enervated.

Fractional power law based nonlocality possess some interesting properties. Tarasov showed that a discrete lattice model can be mapped onto a continua through approximating fractional dispersion relation [14]. In the current work it will be shown that the fractional form of elasticity model has strong resemblance with peridynamic theory. Due to the interesting asymptotic characteristics, fractional power law based constitutive model needs to be studied thoroughly. In literature such thorough investigation has not been noticed. It is realized that this asymptotic feature in both space and frequency domain can be utilized in order to mitigate the wave reflection between strongly–weakly nonlocal models. Besides linking varying nonlocalities, researchers have also been getting interested in linking local and nonlocal models seamlessly for the purpose of concurrent multiscale modeling [15–17]. It was observed that the high frequency components from atomistic scale need to be decayed over the atomistic/classical-continuum handshake region since high frequency short waves do not exist in local models. These high frequency short waves reflect back into the atomistic region and cause artifact in the simulation. In order to have a seamless coupling, the spurious wave reflection can be removed by incorporating artificial stochastic damping [18]. Alternatively, an intrinsic way can be adopted by rectifying the higher frequency components within the handshake region. So, in the current work power law based kernel has been proposed in order to eliminate high frequency waves by utilizing its asymptotic characteristics. It will also be shown that Gaussian kernel is another good candidate for dealing with the wave reflection problem. In order to get an overall picture, power law, Gaussian and polynomial kernels were considered in the current work. For wide range of nonlocality and length scales fractional power law based kernel was proven to be more versatile.

## 2. Wave dispersion in the nonlocal elastic media and prelude to fractional power law

Peridynamics (PD) is considered to be a generalized nonlocal continuum theory. The 1D PD equation of motion is written in Eq. (1). By taking Fourier transformation, Eq. (2) can be obtained. For peristatics, the LHS of Eq. (1) will be zero.

$$\rho\ddot{u} = \int_{-\infty}^{\infty} C(x,x')u(x',t)dx' - P(x,t)u(x,t) + b(x,t), \tag{1}$$

$$-\rho\omega^2\tilde{u}(k,t) = \underbrace{[\mathcal{C}(k) - \mathcal{C}(0)]}_{-\mathcal{M}(k)}\tilde{u}(k,t) + \tilde{b}(k,t). \tag{2}$$

Based on a set of kernel functions $C(x, x')$ the acoustic tensor $\mathcal{M}(k)$ can be written as [19]

$$\text{Classical: } \mathcal{M}(k) = Ek^2, \tag{3}$$

$$\text{Power law: } \mathcal{M}(k) = \frac{E}{l^{2-\alpha}}|k|^{\alpha}, \forall\, k > 0 \text{ and } 0 < \alpha < 2, \tag{4}$$

$$\text{Gaussian-1: } \mathcal{M}(k) = \frac{4E}{l^2}(1 - e^{-\frac{k^2l^2}{4}}), \tag{5}$$

$$\text{Gaussian-2: } \mathcal{M}(k) = Ek^2l^2e^{-\frac{k^2l^2}{4}}, \tag{6}$$

$$\text{Sinusoidal: } \mathcal{M}(k) = \begin{cases} Ek^2l^2, & |kl| < 1, \\ 0, & |kl| > 1. \end{cases} \tag{7}$$

$\mathcal{M}(k)$ for classical continuum (Eq. (3)) is independent of length scale whereas the wave dispersion in nonlocal continua always depends on the length scale parameter $l$. $l$ can be referred to as the cutoff distance for nonlocal interaction. For $l \to 0$, the nonlocal model approaches to be the classical elasticity model. The fractional order in Eq. (4) will introduce nonlocality