Commun Nonlinear Sci Numer Simulat 34 (2016) 130–141

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Stability analysis of electrostatically actuated nano/micro-beams under the effect of van der Waals force, a semi-analytical approach

Amir R. Askari, Masoud Tahani*

Department of Mechanical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

ARTICLE INFO

Article history:
Received 15 May 2014
Revised 21 September 2015
Accepted 21 October 2015
Available online 26 October 2015

Keywords:
Nano/micro-electro-mechanical beams
Pull-in instability
vdW attraction
Homotopy perturbation method
Detachment length
Closed-form expressions

ABSTRACT

The objective of the present paper is to determine pull-in parameters (pull-in voltage and its corresponding displacement) of nano/micro-beams with clamped–clamped, clamped-free, clamped–hinged and hinged–hinged boundary conditions, when they are subjected to the electrostatics and van der Waals (vdW) attractions. The governing non-linear boundary value equation of equilibrium is derived, non-dimensionalized and reduced to an algebraic equation, which describes the position of the maximum deflection of the beam, utilizing the Galerkin decomposition method. The equation which governs on the stability condition of the system is also obtained by differentiating the reduced equilibrium equation with respect to the maximum deflection of the beam. These two equations are solved simultaneously to determine pull-in parameters. Closed-form solutions are provided for cases under electrical loading and vdW attraction alone. The combined effect of both electrostatic and vdW loadings are also investigated using the homotopy perturbation method (HPM). It is found that the present semi-analytical findings are in excellent agreement with those obtained numerically. In addition, it is observed that the present semi-analytical approach can provide results which agree better with available three-dimensional finite element simulations as well as those obtained by non-linear finite element method than other available analytical or semi-analytical findings in the literature. Non-dimensional electrostatic and vdW parameters, which are defined in the text, are plotted versus each other at pull-in condition. It is found that there exists a linear relationship between these two parameters at pull-in condition. Using this fact, pull-in voltage, detachment length and minimum allowable gap of electrostatically actuated nano/micro-beams are determined explicitly through some closed-form expressions.

1. Introduction

Nano/micro-systems have applications in many engineering fields such as communications, automotive and robotics [1]. Nano/micro-electro-mechanical systems (N/MEMS) can be considered as a largest collection of these systems, because of their fast response, low power consumption, reliability and their capability of batch fabrications [2]. Electrically actuated nano/micro-beams represent a major structural component and play a crucial role in many N/MEMS devices [2]. The main components of a typical N/MEMS device can be considered as a fixed electrode and a movable one. The movable electrode can be modeled as an electrically actuated nano/micro-beam or plate which deflects toward the fixed one through the application of an external voltage

* Corresponding author. Tel.: +98 513 8806055; fax: +98 513 8763304.
E-mail address: mtahani@um.ac.ir (M. Tahani).

http://dx.doi.org/10.1016/j.cnsns.2015.10.014
1007-5704/

on the system. The applied voltage has an upper limit in which the electrostatic attraction overcomes the elastic restoring force of the movable electrode. Therefore, the movable electrode suddenly collapses toward the fixed one in this manner. This unstable phenomenon is called pull-in instability. The maximum deflection occurred just before the collapsing of movable electrode is also called pull-in displacement. Pull-in displacement and pull-in voltage, the so-called pull-in parameters, can be considered as the most important parameters in designing electrically actuated nano/micro-beams.

By decreasing the dimensions of electrically actuated systems from micro-scales to nano-scales, the intermolecular surface forces significantly influence on the behaviors of nano/micro-beams. The most important forces at the scale of N/MEMS are the Casimir and vdW attractions. The vdW force arises from the correlated oscillation of the instantaneously induced dipole moments of the atoms placed at the close parallel conductive plates [3]. The vdW force is a short range force in nature, but it can lead to long range effects more than 0.1 μm [4]. The Casimir force can be simply understood as the long range analog of the vdW force, resulting from the propagation of retarded electromagnetic waves [5].

The main purpose of the present paper is to investigate the effect of the vdW attraction on the characteristics of pull-in instability in beam-type N/MEMS. To date, lots of researchers have dealt with the mechanical behavior of electrically actuated nano/micro-beams and plates. A comprehensive review of different models of electrostatically actuated micro-systems are presented by Batra et al. [6]. Herein, some of the most pioneering works in the field of modeling pull-in instability in N/MEMS devices are reviewed.

Nathanson et al. [7] and Taylor [8] simultaneously observed pull-in instability in micro-systems. Pamidighantam et al. [9] studied electrically actuated micro-cantilevers as well as clamped-clamped micro-beams using equivalent spring-mass model. They present a closed-form expression for pull-in voltage and validated their solutions with finite element (FE) simulations provided by Coventorware (CW) commercial software. Their model improved the estimation of previous model presented by Nathanson et al. [7] for pull-in voltage; however, their model represent this value in terms of pull-in displacement which was empirically chosen. Chowdhury et al. [10] also presented a closed-form solution for cantilever micro-beams using another equivalent spring-mass model. They used the third order Maclaurin's series expansion of the electrostatic forcing term to calculate the effective stiffness coefficient of the micro-beam. Kuang and Chen [11] solved the non-linear boundary value equation which governs on the equilibrium of both micro-cantilevers and doubly clamped micro-beams semi-analytically using Adomian decomposition method (ADM). Mojahedi et al. [12] investigated static pull-in instability using the combination of Galerkin's projection method and the HPM. They converted the governing boundary value equation of equilibrium to an algebraic equation using the first linear un-damped mode-shape of an un-deformed micro-beam and solved the resulting non-linear algebraic equation through the HPM. It should be noted that they expanded the electrostatic forcing term about the static deflection of the movable electrode which was updated iteratively by increasing in the value of the input voltage. Therefore, their iterative approach cannot be considered as an analytical procedure. Ramezani et al. [13] proposed a distributed parameter model to study pull-in instability of electrically actuated nano-cantilevers subjected to the vdW and the Casimir forces. They transferred non-linear differential equation of the model into the integral form by using the Green's function of the clamped-free beam and the integral equation was solved using the shape function of the beam deflection. It is to be noted that, their solutions could not satisfy all boundary conditions and suffered from the over-estimation of pull-in voltage. Soroush et al. [14] also investigated the effect of the vdW attraction on pull-in instability of nano/micro-beams using the modified Adomian decomposition method (MADM). Although they could present some expressions for the detachment length and minimum allowable gap of nano/micro-systems, there exists a gap between their findings and those obtained by FE simulations [9].

It is worth noting that the solution of the equation which governs on the static equilibrium has been investigated in previous studies to capture pull-in instability when the slope of deflection-voltage graph reaches infinity. However, in present paper the equilibrium equation and the governing equation on the stability conditions are considered simultaneously. The governing equation of equilibrium is derived using the Euler–Bernoulli beam theory and reduced to an algebraic equation which describes the position of the beam maximum deflection through the Galerkin decomposition method. The stability equation is also obtained by differentiating the reduced equilibrium equation with respect to the maximum deflection of the beam. This set of non-linear algebraic equations is solved analytically for cases under electrical loading alone and vdW attraction alone to present some closed-form expressions for pull-in voltage and displacement as well as the minimum allowable gap and maximum allowable length of the nano/micro-beam. A semi-analytical solution is also performed using the HPM to investigate the interaction between electrical and vdW loadings, where the solution of the system under electrical loading alone is selected as the initial guess to construct the homotopy series solution. It should be noted that unlike the traditional perturbation methods, the HPM does not depend on the assumption of small parameters and can solve strongly non-linear problems [15].

It is found that the present semi-analytical results agree better with available FE predictions than those obtained semi-analytically through beam model in previous studies. In addition, one of the advantages of the present approach is that both pull-in voltage and pull-in displacement are obtained simultaneously which is not possible in previous solution procedures. In other words, the previous studies focused on the solution of the governing equilibrium equation which determined the static deflection in terms of input voltage. This procedure captured pull-in instability form deflection-voltage graph when its slope reaches infinity. However, the present approach provides pull-in parameters straightly. It is also found that the dimensionless pull-in voltage varies linearly versus the non-dimensional vdW parameter. Based on this important finding, some closed-form expressions are introduced to predict pull-in voltage in terms of the system parameters.

It is worth mentioning that the present solution procedure is developed in terms of the nano/micro-beam mode-shapes. Therefore, one just needs to update the mode-shape function to obtain the proper results for any desired boundary conditions.