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Study on Energy Law of Similitude for Laser Propulsion in Repetitively-pulsed Mode

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Abstract

Energy law of similitude for laser propulsion refers to the law that there is an optimum nozzle configuration for the largest value of impulse coupling coefficient at certain incident laser energy. A dimensionless factor combined with incident laser energy, nozzle configuration parameters and working gas parameters is introduced. Energy law of similitude is established by means of theoretical analysis, experimental study and numerical simulation of radiation gas-dynamics. The qualitative results obtained from theoretical analysis are verified by experimental and numerical results. Physical meaning and engineering application of dimensionless factor and energy law of similitude are analyzed. Results indicate that ① impulse coupling coefficient has a maximum value with dimensionless factor of about 0.4; ② impulse coupling coefficient is independent of incident laser energy when dimensionless factor is constant. Conclusions and recognitions acquired in this article can not only present optimum nozzle configurations for the present laser energy level, but also provide a good guide for the optimum nozzle configuration design once the laser energy is amplified to a high level.

Keywords: laser propulsion; law of similitude; repetitively-pulsed mode; conical nozzle; parabolic nozzle; impulse coupling coefficient

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1. Introduction

Nozzle design is very difficult for laser propulsion in repetitively-pulsed mode. Researchers have designed and optimized nozzle configurations numerically and experimentally for constant laser energy such as 100 J[1-8]. However, the optimum nozzle configuration for 100 J will be no more applicable when incident laser energy changes to 1 000 J or even higher energy conditions in the future engineering practice. Moreover, a great number of the existing experimental data cannot provide reference for nozzle design in new conditions.

For traditional rockets, when external gas-dynamics characteristics need to be confirmed, similitude theory can be used to deduce gas-dynamics parameters of prototype with experimental data of scale model obtained from wind tunnel. According to this method, if energy law of similitude for propulsion performance, such as invariance or similitude of impulse coupling coefficient in different laser energy conditions, is established for laser propulsion, the present difficulties in nozzle design can be overcome.

In this article, energy law of similitude for laser propulsion in repetitively-pulsed mode contains three aspects, i.e., similar rules of optimizing nozzle, specific energy and impulse coupling coefficient.

Among different geometrical performances of nozzles for air-breathing laser propulsion in repetitively-pulsed mode, conical nozzles which correspond to linearity characteristics have been mainly discussed. Methods of theoretical analysis, experimental study and numerical simulation of radiation gas-dynamics are used to introduce a dimensionless factor and build energy law of similitude. The dimensionless factor is related to incident laser energy, laser energy deposition rate, nozzle configuration parameters and working gas initial parameters. Furthermore, the conclusion is extended to parabolic nozzles by numerical simulation. The calculated results of the two geometries are compared with each other at the same dimensionless factor. Finally, based on the energy law of similitude, propulsion performance and nozzle configuration have been

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optimized and designed. The models and methods established in this article can be directly used not only in revealing energy conversion for laser propulsion and working mechanism of laser thruster, but also in studying propulsion performance of other complex nozzles, such as nozzle configuration optimization, flight height design, working gas choice, laser parameters design and so on.

2. Analysis on Energy Law of Similitude

2.1. Basic assumptions

When laser energy in the left is deposited at the apex of a conical nozzle with the irradiation of a single laser pulse, a high-temperature and high-pressure plasma region is formed, and then shock wave comes into being (see Fig.1). Strong shock wave flow field formed by one-dimensional spherically-symmetric point-focusing explosion is used to depict self-similar theory[9].

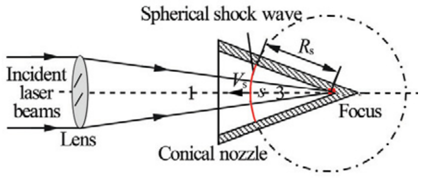

Fig.1 Sketch of a conical nozzle and sphere shock wave.

The disturbed gas density ρ, pressure p and velocity u behind spherical shock wave (i.e. Region 3 in Fig.1) are the functions of dimensionless velocity $\bar{u}$ [10].

$$\frac{\rho}{\rho_s}=\left[\frac{-2(3\gamma-1)\bar{u}+5(\gamma+1)}{-\gamma+7}\right]^{\frac{13\gamma^2-7\gamma+12}{(3\gamma-1)(2\gamma+1)(-\gamma+2)}}\cdot \left[\frac{2\gamma\bar{u}-(\gamma+1)}{\gamma-1}\right]^{\frac{3}{2\gamma+1}}\left[\frac{-2\bar{u}+(\gamma+1)}{\gamma-1}\right]^{\frac{-2}{-\gamma+2}} \tag{1}$$

$$\frac{p}{p_s}=\left[\frac{-2(3\gamma-1)\bar{u}+5(\gamma+1)}{-\gamma+7}\right]^{\frac{13\gamma^2-7\gamma+12}{5(3\gamma-1)(-\gamma+2)}}\cdot \left[\frac{-2\bar{u}+(\gamma+1)}{\gamma-1}\right]^{\frac{-\gamma}{-\gamma+2}}\bar{u}^{6/5} \tag{2}$$

$$\frac{u}{u_s}=\left[\frac{-2(3\gamma-1)\bar{u}+5(\gamma+1)}{-\gamma+7}\right]^{\frac{-(13\gamma^2-7\gamma+12)}{5(3\gamma-1)(2\gamma+1)}}\cdot \left[\frac{2\gamma\bar{u}-(\gamma+1)}{\gamma-1}\right]^{\frac{\gamma-1}{2\gamma+1}}\bar{u}^{3/5} \tag{3}$$

where γ is specific heat ratio, ρ_s, p_s and u_s are the density, pressure and velocity closest behind shock wave (i.e. Region s in Fig.1) respectively, which can be written as

$$\left.\begin{aligned} \rho_s &= \frac{\gamma+1}{\gamma-1}\rho_0 \\ p_s &= \frac{2}{\gamma+1}\rho_0 V_s^2 \\ u_s &= \frac{2}{\gamma+1}V_s \end{aligned}\right\} \tag{4}$$

where ρ_0 is the undisturbed gas density in front of shock wave (i.e. Region 1 in Fig.1), and V_s the propagating velocity of shock wave.

Suppose that cone angle is 2θ and generatrix length (i.e. nozzle length) is R for the conical nozzle, incident laser energy is E_{in}, and laser energy deposition rate is η_{de}. The shock wave propagating distance and velocity are

$$\left.\begin{aligned} R_s &= \left[\frac{25(\gamma+1)(\gamma-1)}{32\pi I_1}\right]^{1/5}\left[\frac{2E_{in}\eta_{de}}{\rho_0(1-\cos\theta)}\right]^{1/5} t^{2/5} \\ V_s &= \frac{2}{5}\left[\frac{25(\gamma+1)(\gamma-1)}{32\pi I_1}\right]^{1/5}\left[\frac{2E_{in}\eta_{de}}{\rho_0(1-\cos\theta)}\right]^{1/5} t^{-3/5} \end{aligned}\right\} \tag{5}$$

$$F = 2\pi\int_0^{R_s}(p-p_0)r\sin^2\theta\,\mathrm{d}r \tag{6}$$

where I_1 is a constant only related to specific heat ratio, r the distance from cone apex ($0 \le r \le R_s$) and F the thrust. Contribution of exhaust and refill stage is ignored here. Analytical equations of flow field parameters are used to deduce propulsion performance parameters, regarding pressure difference of nozzles outside and inside in the axis direction as thrust.

2.2. Dimensionless factor construction

A dimensionless factor $\bar{R}$ is defined as ratio of nozzle length R and radius of a characteristic spherical wave R^*[11], namely

$$\bar{R}=\frac{R}{R^*}=\frac{R}{\left[\dfrac{2E_{in}\eta_{de}}{p_0(1-\cos\theta)}\right]^{1/3}} \tag{7}$$

where p_0 is the undisturbed gas pressure in front of shock wave. $\bar{R}$ is related to incident laser parameters E_{in} and η_{de}, nozzle configuration parameters 2θ and R, and initial parameter p_0 of working gas.

2.3. Impulse and impulse coupling coefficient

Suppose the time when shock wave arrives at the cone exit is t_{arr}. From Eqs.(1)-(7), impulse I and impulse coupling coefficient C_m within 0-t_{arr} can be expressed by cone angle 2θ and dimensionless factor $\bar{R}$ as follows:

$$I=\int_0^{t_{arr}} F\mathrm{d}t \tag{8}$$