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Dynamic optimization of open-loop input signals for ramp-up current profiles in tokamak plasmas[☆]

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ABSTRACT

Establishing a good current spatial profile in tokamak fusion reactors is crucial to effective steady-state operation. The evolution of the current spatial profile is related to the evolution of the poloidal magnetic flux, which can be modeled in the normalized cylindrical coordinates using a parabolic partial differential equation (PDE) called the magnetic diffusion equation. In this paper, we consider the dynamic optimization problem of attaining the best possible current spatial profile during the ramp-up phase of the tokamak. We first use the Galerkin method to obtain a finite-dimensional ordinary differential equation (ODE) model based on the original magnetic diffusion PDE. Then, we combine the control parameterization method with a novel time-scaling transformation to obtain an approximate optimal parameter selection problem, which can be solved using gradient-based optimization techniques such as sequential quadratic programming (SQP). This control parameterization approach involves approximating the tokamak input signals by piecewise-linear functions whose slopes and break-points are decision variables to be optimized. We show that the gradient of the objective function with respect to the decision variables can be computed by solving an auxiliary dynamic system governing the state sensitivity matrix. Finally, we conclude the paper with simulation results for an example problem based on experimental data from the DIII-D tokamak in San Diego, California.

1. Introduction

Fusion is a nuclear reaction in which multiple atomic nuclei collide under very high energy and join together to form a combined atomic nucleus that is lighter than the total mass of the reactants, with the excess mass converted into energy according to Einstein's mass–energy theory. Nuclear fusion reactions occur naturally in the core of the Sun and generate a continuous supply of energy for the universe. Nuclear fusion was first accomplished in the laboratory in the 1950s, and since then has shown considerable promise as a safe, clean and potentially inexhaustible energy production method. As such, it could become the best compromise between nature and the energy needs of mankind.

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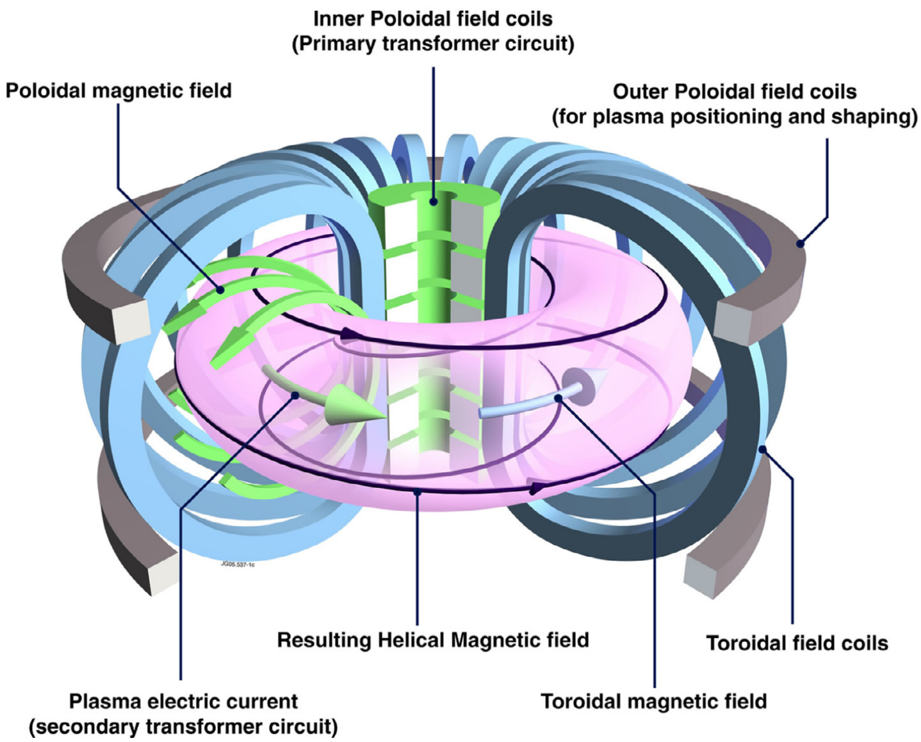

Fig. 1. Schematic of a tokamak chamber and magnetic profile (Source: EFDA-JET).

In the core of the Sun, gravitational forces can compress matter (mostly hydrogen) up to very high densities and temperatures, high enough to ignite fusion reactions. The Sun's gravitational field can maintain the thermonuclear reactions at a controlled and steady rate, by keeping the enormous thermal expansion forces balanced. However, fusion on Earth must be controlled by means other than gravity, as it is impossible to attain in the laboratory density levels near those in the center of the Sun. Hence, for nuclear fusion reactors, it is necessary to work at low gas densities, but at temperatures considerably higher than the temperature in the center of the Sun. At these extremely high temperatures, all matter is in the plasma state, which is considered the fourth state of mass. Fortunately, a torus-shaped device called a tokamak (shown in Fig. 1) can be used to confine the plasma via a helical magnetic field.

Control engineering is considered one of the three critical technologies for achieving viable nuclear fusion power.[1] Accordingly, it has become an important area for multidisciplinary collaboration in the fusion research community. Many exciting research topics are surveyed in the book [2] and two special issues of IEEE Control Systems Magazine.[2] Among various challenging research issues, the control of the current profile in tokamak plasmas is known to be critical to plasma confinement, magnetohydrodynamic stability and effective steady-state operation (e.g., [17,22]).

The evolution in time of the current profile is related to the evolution of the poloidal magnetic flux, which is modeled by the magnetic diffusion equation, a parabolic partial differential equation (PDE) in the normalized cylindrical coordinate system. The problem of manipulating the current profile to achieve high performance while satisfying safety requirements has attracted considerable attention in the literature.

The pioneering work in this area is due to Moreau, who in [15] derived empirical models for current profile evolutions using system identification techniques, and then used these models to synthesize a controller for safety factor profile manipulation. This approach, called the data-driven modeling method, does not rely on first principles to derive the model, but rather involves constructing the model by fitting observed data from experiments (e.g., JT-60 in Japan and DIII-D in the USA [16]), where the observed data is generated by input signals covering a sufficiently broad range of frequencies. As an alternative approach, it is also possible to use PDE models derived from first principles for the control and optimization of internal spatial profiles. This was a popular method in the 1970s and 1980s; one can refer, for example, to the books [4,26]. From the viewpoint of distributed parameter systems governed by PDEs, several advanced control and optimization approaches have been discussed based on new developments in internal diagnostics and plasma actuation. Simplified distributed parameter system models have

[1] Refer to E. Synakowski's presentation titled "Fusion Energy Research: On Our Science, Leverage and Credibility" at the University Fusion Association General Meeting, held during the 51st Annual Meeting of the American Physical Society Division of Plasma Physics (November 2–6, 2009, Atlanta, Georgia, USA).

[2] Refer to papers in the special issues titled "Control of Tokamak Plasmas: Part I" (October 2005) and "Control of Tokamak Plasmas: Part II" (April 2006) in IEEE Control Systems Magazine, organized by A. Pironti and M. Walker.