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Research paper

Nonlinear relative-proportion-based route adjustment process for day-to-day traffic dynamics: modeling, equilibrium and stability analysis

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ABSTRACT

Travelers' route adjustment behaviors in a congested road traffic network are acknowledged as a dynamic game process between them. Existing Proportional-Switch Adjustment Process (PSAP) models have been extensively investigated to characterize travelers' route choice behaviors; PSAP has concise structure and intuitive behavior rule. Unfortunately most of which have some limitations, i.e., the flow over adjustment problem for the discrete PSAP model, the absolute cost differences route adjustment problem, etc. This paper proposes a relative-Proportion-based Route Adjustment Process (rePRAP) maintains the advantages of PSAP and overcomes these limitations. The rePRAP describes the situation that travelers on higher cost route switch to those with lower cost at the rate that is unilaterally depended on the relative cost differences between higher cost route and its alternatives. It is verified to be consistent with the principle of the rational behavior adjustment process. The equivalence among user equilibrium, stationary path flow pattern and stationary link flow pattern is established, which can be applied to judge whether a given network traffic flow has reached UE or not by detecting the stationary or non-stationary state of link flow pattern. The stability theorem is proved by the Lyapunov function approach. A simple example is tested to demonstrate the effectiveness of the rePRAP model.

1. Introduction

Transportation network equilibrium models have been well-known in transportation sciences. The notion of User Equilibrium (UE) proposed by Wardrop [1] is the core of traffic assignment models, which is formulated Beckmann et al. [2] as convex optimization model with the symmetric link performance function. Smith [3] and Dafermos [4] proved that UE with general asymmetric link costs is equivalent to a variational inequality problem. Later, UE has been extended in various aspects, such as the Stochastic UE (SUE) prescribes user's route choice behavior with perception costs errors (e.g., [5,6]), Boundedly Rational UE (BRUE) represents that the route costs of the alternative need to be lower than the costs of the original route minus a threshold, before users switch to the alternative (e.g., [7]). Bi-objective UE (BUE) demonstrates that users can take any optimal path based on two irrelevant nonlinear kinds of costs, such as monetary cost or time cost (e.g., [8,9]). However, these models only focus on the final static equilibrium states, and the process to the final states cannot be described, which can be explained by the day-to-day (DTD) dynamics.

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The aim of DTD dynamics is to capture the evolutionary process of network flows, rather than the final equilibrium states (e.g., [10–16]). As pointed out by Watling and Hazelton [12], the most predominant characteristic to transportation researchers and traffic engineers is the adequate flexibility of DTD approaches, which allows a wide range of behavior rules, levels of aggregation, and traffic modes to be synthesized into a uniform framework. The continuous DTD traffic dynamics was proposed originally and utilized ordinary/partial differentiate equations to describe the fluctuation process itself. Smith [13], Friesz et al. [14], and Zhang and Nagurney [15] proposed the Proportional-Switch Adjustment Process (PSAP), the Network Tatonnement Process (NTP) and the Projected Dynamical System (PDS), respectively. These mechanisms describe the different deterministic traffic assignment models defined on a continuous temporal dimension.

The continuous traffic dynamical systems can be used to analyze the theoretical properties of network traffic flow in evolutionary process. However, Watling and Hazelton [12] pointed out two major weaknesses sustained by continuous DTD traffic modeling methods: (1) continuous-time trip adjustment is not plausible in reality and (2) homogeneous population assumptions in these approaches require additional dispersion modules. In line with the repeated daily principle of driver's route choice behavior, discrete dynamical systems are believed to be the most appropriate for modeling the DTD fluctuation process of network traffic flows [10]. Friesz et al. [14] and Nagurney and Zhang [17] employed particular discretization methods to deal with the dynamical traffic systems in continuous temporal spaces. Moreover, Day-to-day modeling methods can be also applied to modeling the traffic flow evolutionary process after some stochastic capacity-constrained or unexpected disruption traffic network (e.g., [18]). As summarized by Yang et al. [19], all these DTD dynamics modeling are built upon path flow variables and share common property that their stationary points are UE flows. In other words, when the variation of path flows in day-to-day sense are strictly equal to zero flow, the traffic flow must satisfy the principle of classic UE, and vice versa.

Yang et al. [20] investigated the equivalence between day-to-day stationary link flow patterns and UE in the context of traveler's rational route choice behavior. It unveils the essence of UE in the sense of day-to-day link flow pattern, and provides a judgment method whether certain network link flow has achieved equilibrium or not by detecting the stationary or non-stationary states of link flow pattern. Yang et al. [19] shown that five major categories of adjustment mechanisms are Rational Behavior Adjustment Process (RBAP), which are the Simplex Gravity Flow Dynamics (e.g., [21]), the PSAP model (e.g., [22,23]), the Network Tatonnement Process (e.g., [14]), the Projected Dynamical System (e.g., [17]), and Evolutionary Traffic Dynamics (e.g., [24,25]), respectively.

Among these adjustment rules, PSAP has been extended by many researchers (e.g., [10,19,23,26–31]) due to its some instinct advantages (i) traffic flow switches from higher costly path to lower costly path which is consistent with the individual travel behavior pursuing the least cost; (ii) with concise mathematical structure, intuitive route adjustment behaviors, and the PSAP model is the special case of RBAP. Peeta [23] studied the stability issues of the PSAP model by constructed meaningful Lyapunov functions. Recently, Zhang et al. [32] pointed out two behavioral limitations of the PSAP model, i.e., the over-adjustment problem for the discrete PSAP model and the weak robustness problem associated with absolute cost differences. They proposed a nonlinear pairwise swapping dynamic (NPSD) model which can avoid two limitations above. Moreover, they adopt that the Beckmann objective function [2,33] as Lyapunov function to prove the stability of NPSD. However, When link cost function depends on the flows on all links, i.e., the link cost function is non-separable, the Beckmann objective function as Lyapunov function may not be able to prove the stability of NPSD. In addition, NPSD has more complex mathematical structure than PSAP, and lacks intuitive route adjustment behavior rule.

PSAP is (perhaps) the most natural route adjustment process [34,35], it has simple mathematical structure and intuitive route adjustment behavior rule. Thus, this main objective of this paper is to propose a nonlinear relative-Proportion-based Route Adjustment Process (rePRAP) which maintains the advantages of PSAP model and makes up for two behavioral limitations of PSAP model. The proposed model is the special case of the RBAP and the stationary state of the RBAP is consistent with the UE state. The stability results of continuous rePRAP are obtained. The remainder of this paper is formulated as follows. Section 2 introduces the rePRAP model and discusses the relationship between UE and stationary flow patterns of the rePRAP model in detail. In Section 3, the Lyapunov stability analysis of rePRAP is presented. Numerical examples are tested in Section 4. Section 5 concludes this paper.

2. Nonlinear relative-proportion-based route adjustment process

Consider a transportation network represented by a full-connected directed graph $G[N, L]$, where N and L are the set of all nodes and all links, respectively. Let W be the set of origin-destination (OD) pairs, d_w be the fixed travel demand between OD pair $w \in W$, P_w be the set of paths connecting OD pair $w \in W$, f_p^{wt} be the path flow on path $p \in P_w$, $w \in W$, where t is the day index. Denote demand vector, path flow vector, link flow vector and OD-path incidence matrix as $d = (d_w)_{w\in W}$, $f^t = (f_p^{wt})_{p\in P_w}$, $x^t = (x_a^t)_{a\in L}$ and Φ, respectively, then $d = \Phi f^t$. $\dot{f}^t = (\dot{f}_p^{wt})_{p\in P_w}$ is the vector derivative of the path flow vector f^t with respect to t. $C = (C_p^{wt})_{p\in P_w}$ and $c = (c_a^t)_{a\in L}$ are the path cost vector and link cost vector, respectively. Denote link-path incidence matrix as $\Delta = (\delta_{ap})_{a\in L, p\in P_w}$, then $x_a^t = \sum_{w\in W}\sum_{p\in P_w}\delta_{ap} f_p^{wt}$, $\forall a \in L$.

Based on the above-mentioned symbols, assumptions, we first give the assumption about travelers' route choice behaviors, under which we propose the continuous/discrete rePRAP and further prove the rePRAP model is a special case of RBAP, the stationary flow pattern of the rePRAP model is equivalent to UE state in a static transportation network.

For a discrete or day-to-day time-dimensional interval, travelers are assumed to have complete information on previous day's travel costs; they make route choice behavior decisions are based on previous day's information; moreover, travelers