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Nonlinear analysis of a new car-following model accounting for the optimal velocity changes with memory

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ABSTRACT

We, in this study, construct a new car-following model by accounting for the effect of the optimal velocity changes with memory in terms of the full velocity difference (FVD) model. The stability condition and mKdV equation concerning the optimal velocity changes with memory are derived through both linear stability and nonlinear analyses, respectively. Then, the space concerned can be divided into three regions classified as the stable, the metastable and the unstable ones. Moreover, it is shown that the effect of the optimal velocity changes with memory could enhance the stability of traffic flow. Furthermore, the numerical results verify that not only the sensitivity parameter of the optimal velocity changes with memory of driver but also the memory step could effectively stabilize the traffic flow. In addition, the stability of traffic flow is strengthened by increasing the memory step-size of optimal velocity changes and the intensity of drivers' memory with such changes. Most importantly, the effect of the optimal velocity changes with memory may avoid the disadvantage of historical information, which decreases the stability of traffic flow on road.

1. Introduction

Traffic problems play significant impact on the rhythm of human daily life and the routine operation of whole society. To solve the traffic congestion, some kinds of car-following models have been proposed since 1950s. The early car-following models have been proposed by some scholars [1–7]. But these models cannot correctly describe the acceleration in the actual traffic. Therefore, Bando et al. [8] established the expression of acceleration called an optimal velocity (OV) model. Subsequently, many car-following models have been developed on the inspiration of OV model with the consideration of different traffic factors [9–35]. Among them, Helbing and Tilch [9] found that the OV model has high acceleration and unrealistic deceleration in real traffic and proposed a generalized force (GF) model to avoid the defects of the OV model. But the GF model can't properly describe the delay time and the kinematic wave speed at jam density. Furthermore, Jiang [10] et al. developed a full velocity difference (FVD) model by considering positive and negative velocity differences to overcome the shortage of the GF model. Moreover, Nagatani investigated the jamming transition [11,12] and obtained the modified KdV equation [13]. Ge et al. get the KdV and kink–antikink solitons [14] and considered the cooperative driving effects [15].

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Yu et al. [16] further obtained the Kink–antikink density wave in an extended cooperative driving system for car-following model. In addition, Tang et al. [17–20] investigated the effect of road condition and traffic communication factors on traffic flow, such as real-time road conditions, inter-vehicle communication, the reliability of inter-vehicle communication and the driver's forecast effect and so on. Zhu and Yu [21] probed into the effect of a gyroidal road on traffic flow. McCartney [22] presented a discrete time car following model. Yu et al. [23] obtained a new car-following model by taking the prevision driving behavior into account. Zhou et al. [24] proposed a full velocity difference and acceleration model. However, the delay effect and the memory effect have not been addressed yet. In real traffic, driver's delay and memory are inevitable as a car is running on the road. Zhang [25] pointed out that the past information of running process impresses the memory in the driver's mind which plays an important role on traffic behaviors. Recently, several scholars [26–29] have developed certain models concerning the driver's physical delay effect. The results show that it leads to decrease the stability region when driver's physical delay increases. Also, the driver's memory factor has been considered in some extended car-following models [30–32]. It is found that the past information can improve the stability of traffic flow. Furthermore, velocity changes and headway changes with memory feedback [33,34] are also investigated, which shows that the stability of traffic flow can be increased in designing the control strategy for the adaptive cruise control system. However, these models did not involve the optimal velocity changes with memory track, which may heavily influence the traffic flow. In real traffic situation, the optimal velocity changes with memory may estimate the variation of acceleration at the next time step. For those skillful drivers, they possess stronger driving anticipation of optimal velocity and quicker responding ability to the optimal velocity changes with memory than others. Therefore, in this study, we propose a new car-following model considering the effect of optimal velocity changes with memory to explore the influence of optimal velocity changes with memory on the stability of the traffic flow. Meanwhile, both linear stability analysis and nonlinear analysis approaches will be carried out to estimate the impact of optimal velocity changes with memory on the traffic stability and the jamming transition. The numerical simulations will be used to verify such consideration.

2. Model development

Bando et al. [8] firstly proposed an optimal velocity (OV) model, in which the optimal velocity of each vehicle is estimated by the following distance of the preceding vehicle and described in Eq. (1) as below:

$$\frac{dv_n(t)}{dt} = a[V(\Delta x_n(t)) - v_n(t)] \tag{1}$$

where a is the sensitivity coefficient of a driver, $\Delta x_n = x_{n+1} - x_n$ is the headway and $v_n$is the speed. The $V(\Delta x_n(t))$ represents the optimal velocity function adopted as below [8]:

$$V(\Delta x_n(t)) = v_{\max}[\tanh(\Delta x_n(t) - h_c) + \tanh(h_c)]/2 \tag{2}$$

However, the OV model has the drawbacks of high acceleration rate and unrealistic deceleration. To improve such model, Jiang et al. [10] further put forward a full velocity difference (FVD) model with full velocity difference between the following car and leading car as:

$$\frac{dv_n(t)}{dt} = a[V(\Delta x_n(t)) - v_n(t)] + \lambda \Delta v_n(t) \tag{3}$$

where $\Delta v_n(t) = v_{n+1} - v_n$ is the relative speed and λ is a sensitivity coefficient. Also various traffic factors [11–24] are introduced based on OV model. But these previous models cannot describe memory effect. Some references [25–32] indicate that the past information delay decreases the stability of traffic flow. Subsequently, further studies discovered that velocity changes and headway changes with memory can enhance the stability of traffic flow [33,34]. In fact, an individual driver would like approaching individual anticipation of optimal velocity in the light of optimal velocity changes with memory at the next time step. Therefore, we put forward a new optimal velocity changes with memory (OVCM) model for car-following theory as follows:

$$\frac{dv_n(t)}{dt} = a[V(\Delta x_n(t)) - v_n(t)] + \lambda \Delta v_n(t) + \gamma[V(\Delta x_n(t)) - V(\Delta x_n(t - \tau_m))] \tag{4}$$

where τ_m is the memory time step and γ is the sensitivity coefficient of the optimal velocity changes with memory term $[V(\Delta x_n(t)) - V(\Delta x_n(t - \tau_m))]$. When $\gamma = 0$ or $\tau_m = 0$, the form of Eq. (4) reduces to that of FVD model. For simplicity, the Taylor expansion of the variables $\Delta x_n(t - \tau_m)$ is deduced by neglecting the nonlinear terms, i.e.,

$$\Delta x_n(t - \tau_m) = \Delta x_n(t) - \tau_m \frac{d\Delta x_n(t)}{dt} = \Delta x_n(t) - \tau_m \Delta v_n(t) \tag{5}$$

Also, linearizing $V(\Delta x_n(t - \tau))$ and leading to:

$$V(\Delta x_n(t - \tau_m)) = V(\Delta x_n(t)) - \tau_m \Delta v_n(t) V'(\Delta x_n(t)) \tag{6}$$

Then Eq. (4) can be rewritten as follows:

$$\frac{dv_n(t)}{dt} = a[V(\Delta x_n(t)) - v_n(t)] + [\lambda + \gamma \tau_m V'(\Delta x_n)]\Delta v_n(t) \tag{7}$$