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Modeling seasonal measles transmission in China ☆

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ABSTRACT

A discrete-time deterministic measles model with periodic transmission rate is formulated and studied. The basic reproduction number $\mathcal{R}_0$ is defined and used as the threshold parameter in determining the dynamics of the model. It is shown that the disease will die out if $\mathcal{R}_0 < 1$, and the disease will persist in the population if $\mathcal{R}_0 > 1$. Parameters in the model are estimated on the basis of demographic and epidemiological data. Numerical simulations are presented to describe the seasonal fluctuation of measles infection in China.

1. Introduction

Measles is an infection of the respiratory system caused by the paramyxovirus of the genus measles virus [1]. Measles virus (MV) normally grows in the cells of the nasopharynx and lung, and is one of the most contagious human pathogens. It is transmitted by aerosols, infecting a new host via the upper respiratory tract. Eventually, MV infection can spread to many organs including the respiratory [2]. Almost all people without immunity can be infected if exposed to the infectious MV. The virus can be passed from person to person via aerosol droplets containing virus particles, such as those produced by a coughing patient [3].

Measles is an acute, viral infectious disease characterized by high fever, cough, and a maculopapular rash [4]. Severe clinical complications from measles are more common in children under 5 years of age or adults above 20 years old. Most common cause of child death caused by clinical complications from measles are pneumonia and encephalitis [5]. Although effective vaccination measures haves been exerted in many countries, measles is still responsible for about 4% of deaths among children less than the age of 5 worldwide [4]. According to the World Health Organization [6], there are about 30 million people affected each year by measles in the world, of which more than 1200 measles deaths occur every day.

Vaccination has played a major role in preventing the spread of measles infection. In many countries where rubella and mumps infection are also considered to be the major public health burden, measles vaccine is often incorporated with rubella and mumps vaccine as a combined live attenuated measles, mumps, rubella (MMR) vaccine [5]. Current policy is to administer the MMR vaccine to infants between their first and second birthdays [7].

In China, measles is still a public health problem. According to the National Report of Notifiable Communicable Diseases from the Chinese Ministry of Health, the monthly data of measles cases from January 2005 to December 2010 exhibit an obvious periodic pattern on an annual base, with most cases occurring in April or May [8]. This is because measles is a respi-

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ratory infectious disease, the transmission peak is generally attained in the spring. However, with the wide application of measles vaccine, especially the children planned immunity, the seasonal patterns of measles has changed, with higher incidence rate in summer.

It is well-known that many diseases show seasonal behavior [9–11]. Although seasonality may come from various sources, seasonally varying transmission rates and fluctuations in birth rates are two common ones [9,12–14]. In view of the periodic trend of monthly new measles cases and the birth rate in China varies small from 2005 to 2010, this paper will use a model with periodic infection rate $\beta(t)$ to describe the measles transmission in China.

The aim of this study is to investigate the seasonal fluctuation of measles infection by using mathematical modeling approach. By considering the vaccination on newborns, we will formulate an epidemiological model with periodic contact rate to study the threshold dynamics of the model. We apply the model to simulate the seasonal variation of measles in China.

2. Model formulation

We will choose the classical SIR compartment structure to model measles infection though measles experiences an exposure/incubation period. There are several reasons for ignoring the exposed compartment in the model. Firstly, we mainly focus on the seasonal fluctuations of the number of measles cases. Secondly, the contagion of measles in the incubation period is mild. Thirdly, it is not easy to identify an infected individual in the incubation period. Finally, the notifiable disease data are only new measles cases. We assume that the movement of the population in and out of the region is relatively small and can be neglected. We divide the population into three compartments, the susceptibles, the infected and the immunized. Let $S(t)$ denote the number of individuals who are susceptible to measles. Let $I(t)$ denote the number of infective individuals, who are infectious and able to spread the disease by contacting with the susceptible. Let $V(t)$ denote the number of immunized individuals, who have immunity for the rest of their lives due to vaccination or recovery from measles infection. Since the epidemic data are collected or reported in discrete time, and some control measure are carried in discrete time, we formulate a discrete epidemic model with a periodic transmission coefficient $\beta(t)$ to describe the seasonal fluctuation of measles. The standard incidence $\beta(t)SI/N$ is applied because an infectious individual can contact a finite number of persons in one time unit in a large population [15,16]. Since vaccination is an important strategy to control the occurrence and prevalence of measles, we will introduce the vaccination rate p on the newborn children into the model. The movement of the individuals among those compartments is shown in Fig. 2.1. We use a system of difference equations to model the transmission of measles

$$\begin{cases} S(t+1) = S(t) + \Lambda(1-p) - \beta(t)\frac{S(t)}{N(t)}I(t) - \mu S(t), \\ I(t+1) = I(t) + \beta(t)\frac{S(t)}{N(t)}I(t) - (\mu+\gamma)I(t), \\ V(t+1) = V(t) + \Lambda p + \gamma I(t) - \mu V(t), \quad t = 0,1,2,\ldots \end{cases} \tag{2.1}$$

where Λ is the recruitment rate, μ is the natural death rate, p is the vaccination rate on the newborn children, and γ is the recovery rate of the infective. $\beta(t)$ is the periodic transmission rate, satisfying $\beta(t) = \beta(t+\omega)(\omega \in \mathbb{Z}_+)$ and $0 < \beta(t) < 1$ for all $t \geqslant 0$. All parameters in model (2.1) are positive and satisfy

$$0 < p < 1, \quad 1 - \mu - \beta(t) > 0, \quad \mu + \gamma < 1.$$

For given initial data $(S(0), I(0), V(0)) \in \mathbb{R}^3_+$, the solution of model (2.1) exists and has nonnegative components for all $t \geqslant 0$, and thus the model is well posed. Let $N(t) = S(t) + I(t) + V(t)$, then

$$N(t+1) = \Lambda + (1-\mu)N(t), \text{ or } N(t+1) = \Lambda\frac{1-(1-\mu)^t}{\mu} + (1-\mu)^t N(0).$$

It is easy to see that $N(t) \to \Lambda/\mu$ as $t \to \infty$, and hence the feasible region

$$\Omega = \{(S, I, V) : S, I, V \geqslant 0, S + I + V \leqslant \Lambda/\mu\}$$

is a positively invariant set for model (2.1).

Model (2.1) always has a disease-free equilibrium $E_0 = (S_*, 0, V_*)$, where $S_* = \frac{\Lambda(1-p)}{\mu}$ and $V_* = \frac{\Lambda p}{\mu}$. Linearizing (2.1) at E_0, we get the equation for I

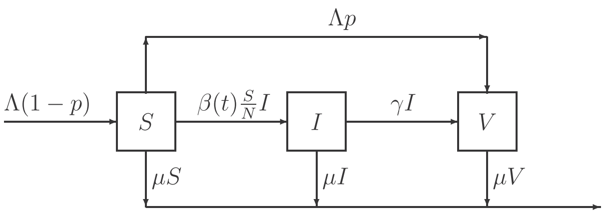

Fig. 2.1. The flow diagram of the measles transmission.