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Model reference adaptive control in fractional order systems using discrete-time approximation methods

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ABSTRACT

In this paper, model reference control of a fractional order system has been discussed. In order to control the fractional order plant, discrete-time approximation methods have been applied. Plant and reference model are discretized by Grünwald–Letnikov definition of the fractional order derivative using "Short Memory Principle". Unknown parameters of the fractional order system are appeared in the discrete time approximate model as combinations of parameters of the main system. The discrete time MRAC via RLS identification is modified to estimate the parameters and control the fractional order plant. Numerical results show the effectiveness of the proposed method of model reference adaptive control.

1. Introduction

In "model reference control" in general, the objective is to have a system that follows a certain system as a reference model. In the real world, usually adaptive controller is designed to achieve this goal, despite the uncertainties that may occur in the parameters of the system [1]. Model reference adaptive control (MRAC) is developed in both continuous-time [2] and discrete-time systems [3,4] in the form of general controller and model following methods. Reference model following can be done in both "Indirect Method" or "Direct Method" called "Projection Method" in some texts [5].

Fractional calculus has a history of about 300 years, but not much attention had been done in that time. Recently there are a lot of works and applications which can be found about fractional calculus [6–10]. Fractional calculus was developed and generalized in recent years [11,12], and also was used more and more in general applications. As an example, modeling of viscoelastic materials has been done in many old and recent works using fractional order derivatives [13–16]. Also fractional calculus was used in anomalous diffusion in [17].

Fractional equations were recently used in "control" because of more flexibility and also memory-dependent property. For example, PID controller in fractional space called $PI^{\lambda}D^{\mu}$ or Fractional Order PID (FO-PID) has 5 parameters (instead of 3) to be tuned [18,19]. So many tuning algorithms for FO-PIDs were developed later [20–23] and FO-PIDs came to real-world applications [24,25]. Fractional order controllers have joined in many branches of classical control such as sliding mode control [26], back-stepping method [27] or MRAC.

Lyapunov stability method and the MIT rule were used to generate adaptation laws for MRAC in Integer-Order systems in classical texts [2]. Also a fractional adjustment rule in MRAC was studied in [28,29], which both utilized fractional order adaptation laws to achieve better performance in model following and parameter estimation, but no stability analysis

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was included. In those works some modifications are applied to the MIT rule in regular systems. However no stability analysis has been presented for the MIT rule in fractional or even integer order systems. Stability analysis in FO-MRAC was done in [30] using the frequency distributed fractional integrator model [31]. In [32] a method for adaptive feedback was proposed by discretization of a fractional dynamic system in which the system and the controller were linear. FO-MRAC was used in many applications like adjusting controller parameters [33], control of autonomous guided vehicles (AGV) [34], hydraulic driven flight motion simulator [35], boiler burning system [36]. Also in chaos synchronization problems, fractional-order adjustment rules were used in some recent works [37–39].

In the present paper, first of all we have discussed on some preliminaries about definitions of fractional derivative. Then a discretization formula has been proposed based on the definitions. In Section 3, general adaptive controller for discrete systems has been studied and adaptive controller for the approximated fractional order system is designed in the next section. Some simulations are performed and the results are presented in Section 5. The main point of this paper is the use of approximation of discretization methods, to design an adaptive controller for fractional order dynamical systems. The presented paper, proposes a method that can have proper stability analysis and can be applied generally in "model reference adaptive control". One can use this method to make an integer order system follow a fractional order dynamics, and vice versa.

2. Preliminaries

Fractional order calculus is a generalized version of integer order calculus. The theory of derivatives of non-integer order was first mentioned by Leibnitz in 1965. After that, more exact definitions were generated by Liouville, Grünwald, Letnikov and Riemann. The integro-differential operator is shown by ${}_{t_0}D_t^{\alpha}$.

2.1. Definitions

Common formulations for fractional derivatives are as follows.

Definition 1 (*Riemann–Liouville fractional derivative [40]*). The Riemann–Liouville fractional derivative is defined as:

$$ {}_{t_0}^{RL}D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(-\alpha)} \int_{t_0}^{t} (t-\tau)^{-\alpha-1} f(\tau)d\tau & \alpha < 0 \\ f(t) & \alpha = 0, \\ D^n[{}_{t_0}D_t^{\alpha-n} f(t)] & \alpha > 0 \end{cases} \quad (1) $$

where $n-1 \leqslant \alpha < n$ and $\Gamma(\cdot)$ is the standard gamma function, $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$.

Definition 2 (*Caputo fractional derivative [40]*). The Caputo fractional derivative is defined as:

$$ {}_{t_0}^{GL}D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau & n-1 < \alpha < n \\ D^n f(t) & \alpha = n \end{cases} \quad (2) $$

The Caputo fractional derivative was almost used in engineering problems, because in stating the initial conditions, derivatives are appeared on integer points, so it has physical implementation.

Definition 3 (*Grünwald–Letnikov fractional derivative*). The Grünwald–Letnikov operator is defined as:

$$ {}_{t_0}^{GL}D_t^{\alpha} f(t) = \lim_{h\to 0} \frac{(\Delta_h^{\alpha} f)(t)}{h^{\alpha}} = \lim_{\substack{h \to 0 \\ Nh = x}} \frac{1}{h^{\alpha}} \sum_{k=0}^{N} (-1)^k \binom{\alpha}{k} f(t-kh) \quad (3) $$

This definition of fractional derivative leads to Riemann–Liouville definition when we perform limit operation. The proof of this can be seen in [40].

Finite Grünwald–Letnikov operator can be defined if we do not perform the limit operation $h \to 0$ and it is denoted by ${}_F^{GL}D^{\alpha}$.

Definition 4. A general dynamical system in fractional calculus is defined as:

$$ F\left(t, y(t), {}_{t_0}^{*}D_t^{\alpha_1} y(t), {}_{t_0}^{*}D_t^{\alpha_2} y(t), \ldots, {}_{t_0}^{*}D_t^{\alpha_n} y(t)\right) = g(t) \quad (4) $$

where $\alpha_1 < \alpha_2 < \cdots < \alpha_n$, $F(t, y_1, \ldots, y_n)$ and $g(t)$ are real known functions. The state-space implementation of system (4) can be written as:

$$ {}_{t_0}^{*}D_t^{\alpha_i} x_i = f_i(t, x_1, x_2, \ldots, x_n), \quad x_i(0) = x_{i0}, \quad i = 1, 2, \ldots, n, \quad (5) $$