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Novel coupling scheme to control dynamics of coupled discrete systems

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ABSTRACT

We present a new coupling scheme to control spatio-temporal patterns and chimeras on 1-d and 2-d lattices and random networks of discrete dynamical systems. The scheme involves coupling with an external lattice or network of damped systems. When the system network and external network are set in a feedback loop, the system network can be controlled to a homogeneous steady state or synchronized periodic state with suppression of the chaotic dynamics of the individual units. The control scheme has the advantage that its design does not require any prior information about the system dynamics or its parameters and works effectively for a range of parameters of the control network. We analyze the stability of the controlled steady state or amplitude death state of lattices using the theory of circulant matrices and Routh–Hurwitz criterion for discrete systems and this helps to isolate regions of effective control in the relevant parameter planes. The conditions thus obtained are found to agree well with those obtained from direct numerical simulations in the specific context of lattices with logistic map and Henon map as on-site system dynamics. We show how chimera states developed in an experimentally realizable 2-d lattice can be controlled using this scheme. We propose this mechanism can provide a phenomenological model for the control of spatio-temporal patterns in coupled neurons due to non-synaptic coupling with the extra cellular medium. We extend the control scheme to regulate dynamics on random networks and adapt the master stability function method to analyze the stability of the controlled state for various topologies and coupling strengths.

1. Introduction

The control of chaos in nonlinear systems has been an active field of research due to its potential applications in many practical situations where chaotic behavior is not desirable. Since the time of Grebogi–Ott–Yorke [1] several methods were proposed to control chaotic dynamics to desired periodic states or to stabilize unstable fixed points [2] of the system. Recently these techniques have been extended to control spatio-temporal chaos in spatially extended systems [3]. They are useful in many applications to control dynamics in plasma devices and chemical reactions [4,5] and to reduce intensity fluctuations in laser systems [6,7]. We note that control is important for emergence of regulated and sustainable phenomena in biological systems, for example to ensure stability of signal-off state in cell signaling networks which is desirable to

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prevent autoactivation [8]. Also in general, chaotic oscillations can degrade the performance of engineered systems and hence effective and simple control strategies have immense relevance in such cases.

The dynamics of spatially extended systems can be modeled in a very simple but effective manner by using coupled map lattices (CML) introduced by Kaneko [9,10]. This approach forms an efficient method to coarse grain the local dynamics in such systems and has been used to understand complex natural phenomena. The dynamical states in such systems are extremely rich and varied, including spatiotemporal chaos, regular and irregular patterns, traveling waves, spiral waves, etc. [11]. In the case of coupled map lattices, methods for control and synchronization reported earlier are mostly based on nonlinear feedback control [12,13], constant and feedback pinning, etc. [14,15]. Recently coupled chaotic maps have been shown to stabilize to a homogeneous steady state due to time delay in coupling [16] and in the presence of random delays [17]. Similarly, using multiple delay feedback, unstable steady states are shown to stabilize in chaotic electronic circuits [18]. Under threshold activated coupling at selected pinning sites, chaotic neuronal maps are reported to stabilize to regular periodic patterns [19]. In one way coupled map lattice decentralized delayed feedback can introduce control of chaos [20]. We note that most of the control schemes like feedback and pinning, the control units are often derived from the dynamics of the system and as such must be designed specific to each system. Also delay feedback makes the system higher dimensional from the analysis point of view and requires careful choice of delay and its implementation for achieving control. However in many applications for the practical implementation of the control scheme, it is desirable to have a general scheme requiring minimum information about the system to be controlled.

In this paper, we introduce a coupling scheme, which can be applied externally to the system and does not require a priori information about the system. As such it is very general and effective and can be implemented easily for control of spatiotemporal dynamics and patterns on coupled discrete systems. We note that in the context of continuous systems, one of the methods recently reported to induce amplitude death or steady state in coupled systems is coupling to an external damped system referred to as environmental coupling or indirect coupling [21–23]. This method has been successfully implemented using electronic circuits [24] and applied for controlling dynamics of single systems [25] and systems with bistability [26] as linear augmentation. These methods in general involve a single external system to control the dynamics of coupled continuous systems. The present paper extends this particular method in two ways: the method proposed is for controlling dynamics in discrete systems and the external control system is a spatially extended system. We find that the spatial extension has its own advantages and relevance as a method for suppressing dynamics.

The control system in our scheme is designed as an external lattice of damped discrete systems such that without feedback from the system, this control lattice stabilizes to a global fixed point. When system and control lattices are put in a feedback loop, the mutual dynamics works to control their dynamics to a homogeneous steady state or periodic state. We illustrate this using logistic map and Henon map as site dynamics. The analysis is developed starting with a single unit of the interacting system and the control, which is then extended to connected rings of systems, interacting 2-dimensional lattices and random networks. This 'bottom up approach' is chosen, not only because it gives clarity in describing the mechanism but also because the cases of even single system or rings are not studied or reported so far for discrete systems. We analyze the stability of the coupled system and control lattices using the theory of circulant matrices and Routh–Hurwitz criterion for discrete systems. Thus we obtain regions in relevant parameter planes that correspond to effective control. We also report results from detailed numerical simulations that are found to agree with that from the stability analysis.

In particular cases, we obtain control even when the units in the system lattice are uncoupled. So also, uncoupled units in the control lattice can control the coupled system lattice. Moreover by tuning the parameters of the control lattice, we can achieve control to regular periodic patterns on the system lattice. Recently, chimera states in a 1-d lattice with nonlocal couplings have been realized using liquid crystal light modulator [27]. We show how our scheme can control the chimera states in this system.

The extended and external control system introduced here can model an external medium effectively in controlling the dynamics of real world systems. As an example, we show how the dynamical patterns and undesirable excitations produced by coupled neurons can be controlled due to interaction with the extra cellular medium. In the end we extend the scheme to control the dynamics of discrete time systems on a random network. In this case, the stability of the controlled state is analyzed using master stability function method and supported by direct numerical simulations.

2. Control scheme for 1-d coupled map lattice

In this section we introduce our scheme for controlling a 1-d CML of size N by coupling with an equivalent lattice of damped systems. The dynamics at the i^{th} node of the system lattice constructed using the discrete dynamical system, $\mathbf{x}(n+1) = \mathbf{f}(\mathbf{x}(n))$, with time index n, is given by:

$$\mathbf{x}_i(n+1) = \mathbf{f}(\mathbf{x}_i(n)) + D\zeta(\mathbf{f}(\mathbf{x}_{i-1}(n)) + \mathbf{f}(\mathbf{x}_{i+1}(n)) - 2\mathbf{f}(\mathbf{x}_i(n))) \tag{1}$$

Here $\mathbf{x}\epsilon\mathbb{R}^m$ is the state vector of the discrete system, $\mathbf{f} : \mathbb{R}^m \to \mathbb{R}^m$ is the nonlinear differentiable function. Also, D represents the strength of diffusive coupling and ζ is a $m \times m$ matrix, entries of which decide the variables of the system to be used in the coupling. The dynamics of a damped discrete map $z(n)$ in the external lattice is represented as:

$$z(n+1) = kz(n) \tag{2}$$