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Analysis of a novel two-lane lattice model on a gradient road with the consideration of relative current

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ABSTRACT

In this paper, a novel hydrodynamic lattice model is proposed by considering of relative current for two-lane gradient road system. The stability condition is obtained by using linear stability theory and shown that the stability of traffic flow varies with three parameters, that is, the slope, the sensitivity of response to the relative current and the rate of lane changing. The stable region increases with the increasing of one of them when another two parameters are constant. By using nonlinear analysis, the Burgers, Korteweg–de Vries, and modified Korteweg–de Vries equations are derived to describe the phase transition of traffic flow. Their solutions present the density wave as the triangular shock wave, soliton wave, and kink–antikink wave in the stable, metastable, and unstable region, respectively, which can explain the phase transitions from free traffic to stop-and-go traffic, and finally to congested traffic. To verify the theoretical results, a series of numerical simulations are carried out. The numerical results are consistent with the analytical results. To check the novel model, calibration are taken based on the empirical traffic flow data. The theoretical results and numerical results show that the traffic flow on the gradient road becomes more stable and the traffic congestion can be efficiently suppressed by considering the relative current and lane changing, and the empirical analysis shows that the novel lattice model is reasonable.

1. Introduction

Traffic congestion has attracted wide attention because it produces significantly negative effect on urban residents' daily lives. Although increasing road supply is a practical way to alleviate traffic congestion, it does not always work due to limited budget or space. How to relieve congestion efficiently needs a set of scientific methods. To this aim, the first thing is to understand the formation mechanism of traffic congestion. Mathematical modeling is the effective method to gain a good understanding of traffic congestion formation. In the past decades, numerous traffic flow models were developed such as hydrodynamics models [1], car-following models [2–12], cellular automation models [13], and lattice models [14–47].

The traffic flow lattice model [14], as the simplified version of the macroscopic hydrodynamics model, was initially proposed by Nagatani to describe the dynamical phase transitions on the freeway. The basic idea behind such models is that drivers adjust their velocity according to the observed headway. The lattice model incorporates the ideas of car-following models, so it has aroused widespread attention. The lattice model is easier to analyze and simulate because it is described by both continuous temporal variable and discrete spatial variable on one-dimensional lattice. Soon thereafter, Nagatani presented some improved

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lattice models such as triangular lattice model [16], two-dimensional lattice model [17], high-dimensional lattice model [18], and analyzed jamming transition in these models [14–19].

Although the original lattice model is shown to have the universal structure in describing many properties of the traffic flow, many approaches to extend the model toward more realistic lattice models have been pursued. Here, we make a brief review to lattice models. Xue presented an extended model by considering the next-nearest-neighbor flow [20]. Li et al. and Tian et al. investigated the effect of relative current difference in the lattice model [22,28]. Zhu et al. and Tian et al. extended the lattice model with consideration of optimal current difference [27,29]. By considering drivers' factor, Ge et al. considered the “backward looking” [31] and driver's physical delay effect [34] in the lattice hydrodynamic model. They also carried out the theoretical analysis of the lattice models [32,33]. Sun et al. presented a new lattice hydrodynamic traffic flow model with the consideration of multi-anticipation effect [26]. Cheng et al. [24] made a theoretical analysis of the anticipation lattice models. Peng considered a driver's memory [39]. Zhang et al. presented an extended two-lane traffic flow lattice model with driver's delay time [44]. For two-lane traffic system, Tang et al improved two-lane lattice model by introducing a new function to govern the flow shift between two lanes [21]. Peng et al. considered the driver anticipation effect [41], multi-anticipation effect [42] and the honk effect [43]. Wang et al. considered driver's delay time [45] and introduced density difference into the lattice model [46,47]. Gupta et al. considered driver's anticipation effect in sensing relative flux [36] and density difference [37].

Only flat roads were considered in the models mentions above. But in real traffic, the road may not always be flat. By considering varying road condition, Tang et al. proposed a car following model on a gradient road [10]. Chen et al. presented an extended car-following model with consideration of the gravitational force and developed a new macroscopic model taking the slope effects into account [25]. Zhu et al. proposed a new lattice model on a gradient highway [30]. Peng et al. proposed an non-lane-based lattice hydrodynamic model [38]. Ge et al. present a lattice model for bidirectional pedestrian flow on gradient road [34]. To our knowledge, until now, the effect of the relative current have not been taken into account in the lattice model on two-lane gradient road. In real traffic, relative current plays an important role on the decision of the accelerating or decelerating, so it should not be overlooked in modeling traffic flow on the gradient road. Based on this idea, we propose a novel lattice model on two-lane gradient road by taking relative current into account. The paper is organized as follows. In Section 2, we review the original lattice hydrodynamic model and present the novel lattice model. In Section 3, we discuss the linear stability of the novel model. In Section 4, the Burgers, Korteweg–de Vries (KdV), and modified Korteweg–de Vries (mKdV) equations are derived in three types of traffic flow regions by using nonlinear analysis. Numerical simulations are carried out in Section 5. Empirical analysis was taken in Section 6 based on the real traffic flow data. The conclusion is drawn in Section 7.

2. Model

2.1. Nagatani's models

In 1998, Nagatani proposed a lattice model [14] with dimensionless space to describe the traffic phenomena on a single lane, which is described by the following partial equations:

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0 \tag{1}$$

$$\partial_t(\rho_j v_j) = a[\rho_0 V(\rho_j + 1) - \rho_j v_j] \tag{2}$$

where j indicates site j on the one-dimensional lattice, ρ_j and v_j represent, respectively, the local density and velocity on site j at time t. ρ_0 is the average density, a is the sensitivity of drivers.

The optimal velocity function, $V(\cdot)$, which is given by Bando et al. [2], is adopted in Eq. (2) as follows:

$$V(\rho) = \frac{v_{max}}{2}\left[\tanh\left(\frac{1}{\rho} - \frac{1}{\rho_c}\right) + \tanh\left(\frac{1}{\rho_c}\right)\right]. \tag{3}$$

where v_{max} is the maximum velocity, and ρ_c is safety density which is the inverse of the safety distance h_c, that is, $\rho_c = 1/h_c$. For the symmetry of density, Eq. (3) can be rewritten by

$$V(\rho) = \tanh\left(\frac{2}{\rho} - \frac{\rho}{\rho_0^2} - \frac{1}{\rho_c}\right) + \tanh\left(\frac{1}{\rho_c}\right). \tag{4}$$

This function has the turning point (inflection point) at $\rho = \rho_c$, when $\rho_c = \rho_0$.

Considering lane changing, Nagatani [17] extended the single-lane model to the two-lane traffic. Fig. 1 shows the schematic model of traffic flow on a two-lane hightway. Without lane changing, the continuity Eq. (1) satisfies from the conservation law of density at site j. However, if the density at site $j-1$ on the second lane is higher than that at site j on the first one, the lane changing occurs from the second lane to the first lane with rate $\gamma\left|\rho_0^2 V'(\rho_0)\right|(\rho_{2,j-1}(t) - \rho_{1,j}(t))$, where $\rho_{1,j}(t)$ and $\rho_{2,j}(t)$ are the densities on the first and second lanes, and dimensionless γ is the rate constant coefficient. Similarly, if the density at site j on the first lane is higher than that at site $j+1$ on the second lane, the lane changing occurs from the first lane to the second lane with the rate $\gamma\left|\rho_0^2 V'(\rho_0)\right|(\rho_{1,j}(t) - \rho_{2,j+1}(t))$. Thus, the continuity equation on the first lane and the second lane are, respectively,

$$\partial_t \rho_{1,j} + \rho_0(\rho_{1,j} v_{1,j} - \rho_{1,j-1} v_{1,j-1}) = \gamma|\rho_0^2 V'(\rho_0)|(\rho_{2,j+1} - 2\rho_{1,j} + \rho_{2,j-1}), \tag{5}$$