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Modulus synchronization in a network of nonlinear systems with antagonistic interactions and switching topologies

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ABSTRACT

This paper studies the collective behavior in a network of nonlinear systems with antagonistic interactions and switching topologies. The concept of modulus synchronization is introduced to characterize the case that the moduli of corresponding components of the agent (node) states reach a synchronization. The network topologies are modeled by a set of directed signed graphs. When all directed signed graphs are structurally balanced and the nonlinear system satisfies a one-sided Lipschitz condition, by using matrix measure and contraction theory, we show that modulus synchronization can be evaluated by the time average of some matrix measures. These matrices are about the second smallest eigenvalue of undirected graphs corresponding to directed signed graphs. Finally, we present two numerical examples to illustrate the effectiveness of the obtained results.

1. Introduction

In the last decades, the concepts of consensus and synchronization are very popular in systems and control, thanks to many applications in physics, biology, society, and engineering. Consensus and synchronization mean that all agent states achieve agreement about some variables of interest. In consensus literature [1–4], the emphasis is on the communication constraints (time invariant or time varying graph) and the dynamic of each agent is usually very simple, e.g. integrator or linear time invariant system. However, in synchronization literature [5–8], the emphasis is on the dynamic of each agent rather than on the communication limitations. Meantime, as pointed out in [9], coordination problems encountered in the engineering world can often be rephrased as consensus or synchronization problems in which both the dynamic of each agent and the communication constraints play an important role. Designing interconnection control laws to ensure synchronization in a network of nonlinear systems has attracted much attention in recent years (see, e.g. [10–12]).

Designing distributed control laws to achieve consensus (synchronization) collaborative relationship among agents is a common assumption in most of the literature. Mathematical formulation of collaborative relationships can be presented as a nonnegative graph (positive edges). However, competition (antagonists) is another inherent relationship among agents in natural and engineering systems, such as competing species and competitive cellular neurons [13,15,16], social networks [14,18]. When there coexist both collaborative and competitive (antagonistic) relationships in a network, it is not expected to achieve consensus. Recently, bipartite consensus was considered in some papers [17,19–25], where bipartite consensus means that all agents converge to a value which is the same for all in modulus but not in sign. In [17], Altafini studied the collective behavior in a

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network of integrators over signed graph, and proved that the network admits a bipartite consensus solution if and only if the signed graph is structurally balanced. This result has been extended to general linear multi-agent systems [20,24] and dynamic output feedback control [21], where each node is modeled by a linear time-invariant (LTI) system. Under a weak connectivity assumption that the signed network has a spanning tree, the papers [22,23] obtained some sufficient conditions for bipartite consensus of multi-integrator systems on directed signed networks. In addition, the authors of [19] studied the bipartite flock control problem in multi-integrator systems. It is well known that almost all real systems are nonlinear. Our recent work [25] studied the bipartite synchronization in a network of nonlinear systems using contraction approach. Meng et al. [26] introduced a concept of modulus consensus in the sense that the moduli of the node states reach a consensus, and studied the modulus consensus in a network of discrete-time linear systems with antagonistic interactions and switching topology. We can see that if a network is bipartite consensus, then it is also modulus consensus.

In this paper, we study the collective behavior in a network of nonlinear systems with antagonistic interactions and switching directed topologies. Mathematical formulation of the network topology can be presented as a set of directed signed graphs. We introduce the concept of modulus synchronization in the sense that the moduli of corresponding components of the agent states reach synchronization. By using matrix measure, we obtain some sufficient conditions in form of matrix measures about the second smallest eigenvalue of undirected graphs corresponding to directed signed graphs, such that the network achieves modulus synchronization. For general switching signals, the modulus synchronization can be easily evaluated by the average of matrix measures of each switching interval. In particular, for periodic signals, the modulus synchronization can be easily evaluated by the average of matrix measures in a periodic interval. Two numerical examples are presented to illustrate the effectiveness of the obtained results.

The paper is organized as follows. Section 2 presents some notations and problem statement. Section 3 investigates the modulus synchronization in a network of nonlinear systems. Section 4 shows some numerical examples to verify the obtained results. Section 5 summarizes our conclusions and describes future work. Appendix A presents some definitions and facts about signed graphs. Appendix B recalls some facts about matrix measure and contraction theory.

2. Notations and problem statement

2.1. Notations

The following notations are used throughout this article. The notation $|x|$ denotes the absolute value of real number x. The symbol I_N denotes the N-dimensional identity matrix, and the operator $\otimes$ denotes the Kronecker product. For a Laplacian matrix $L \in \mathbb{R}^{n\times n}$, we use the notations $\lambda_1(L) \le \lambda_2(L) \le \cdots \le \lambda_n(L)$ to denote the eigenvalues of L.

2.2. Problem statement

Consider a network with N nonlinear dynamical systems described by

$$\dot{x}_i(t) = f(x_i) + Bu_i, \tag{1}$$

where $i = 1, 2, \dots, N$, $x_i \in \mathbb{R}^n$ is the state of the ith agent, $u_i \in \mathbb{R}^m$ is the input, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth odd function, and $B \in \mathbb{R}^{n\times m}$ is a constant matrix. We assume that there coexist collaborative and antagonistic relationships in this network, and the topology of the network is switching between p directed signed graphs $\mathcal{G}(A^i)$, $i = 1, \dots, p$, that is, the interactions between agents are

$$u_i = c\sum_{j=1}^{N}\left|a_{ij}^{\sigma(t)}\right|\left(\mathrm{sgn}(a_{ij}^{\sigma(t)})x_j - x_i\right), \tag{2}$$

where $i = 1, 2, \dots, N$, c is the overall coupling strength, sgn is signature function, $\sigma(t) : [0, \infty) \to \mathcal{P} = \{1, 2, \dots, p\}$ is a switching signal, which is a piecewise and right continuous constant function whose switching instants $\{t_i : i = 0, 1, \dots\}$ satisfies $t_{i+1} - t_i \ge T_1 > 0$, $\forall i \ge 0$, $A^{\sigma(t)} = (a_{ij}^{\sigma(t)}) \in \mathbb{R}^{N\times N}$ is the adjacency matrix associated with the switching function $\sigma(t)$. The combination of (1) and (2) becomes

$$\dot{x}_i(t) = f(x_i) + Bc\sum_{j=1}^{N}\left|a_{ij}^{\sigma(t)}\right|\left(\mathrm{sgn}(a_{ij}^{\sigma(t)})x_j - x_i\right), \tag{3}$$

where $i = 1, 2, \dots, N$.

Definition 1. The network (3) is said to achieve modulus synchronization if $\lim_{t\to\infty}|x_{1j}| = \lim_{t\to\infty}|x_{2j}| = \cdots = \lim_{t\to\infty}|x_{Nj}|$, where $j = 1, \dots, n$.

Remark 1. Note that if f is a linear function, then the concept "modulus synchronization" is same to the concept "modulus consensus" in [26]. As pointed out in [9], the researchers often use the concept "synchronization" in complex networks in which the dynamical behavior of each agent is very complex. Thus we use the concept "modulus synchronization" in Definition 1. The concept "synchronization" in [5,9] means that states of the agents converge to the same value. The concept "bipartite synchronization" in [25] means that there exists $j \in \{1, \dots, n\}$ such that $\lim_{t\to\infty}|x_{1j}| = \lim_{t\to\infty}|x_{2j}| = \cdots = \lim_{t\to\infty}|x_{Nj}| \ne 0$. Hence, if