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Higher-order semirational solutions and nonlinear wave interactions for a derivative nonlinear Schrödinger equation

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ABSTRACT

We present the semirational solution in terms of the determinant form for the derivative nonlinear Schrödinger equation. It describes the nonlinear combinations of breathers and rogue waves (RWs). We show here that the solution appears as a mixture of polynomials with exponential functions. The k-order semirational solution includes $k-1$ types of nonlinear superpositions, i.e., the l-order RW and (k-l)-order breather for $l = 1, 2, \ldots, k-1$. By adjusting the shift and spectral parameters, we display various patterns of the semirational solutions for describing the interactions among the RWs and breathers. We find that k-order RW can be derived from a l-order RW interacting with $\frac{1}{2}(k-l)(k+l+1)$ neighboring elements of a $(k-l)$-order breather for $l = 1, 2, \ldots, k-1$.

1. Introduction

As two typical nonlinear waves, the breather and rogue wave (RW) have become the very interesting topic in both experimental and theoretical studies. The breathers develop owing to the instability of small amplitude perturbations which may grow in size to disastrous proportions [1]. Up to now, there mainly exist two types of breathers such as longitudinal breathers [Kuznetsov-Ma breathers (KMBs)] [2] and transverse breathers [Akhmediev breathers (ABs)] [3]. KMBs are periodic in time and localized in space, while ABs are periodic in space and localized in time [2,3]. When the periods of ABs or KMBs are taken to be infinite, they become the rational solution-the Peregrine soliton (PS), which serves as the prototype of RW [4]. RWs describe a unique event that seems to appear from nowhere and disappear without a trace, the height or steepness of which can usually attain two or three times greater than the average wave state [5]. In contrast to ordinary solitons which enjoy a strong stability, RWs are the localized structures with the instability and unpredictability [5,6]. The generation mechanism of the RW is related to the Benjamin–Fier instability [7], which is also referred to as the modulation instability. A true RW can be generated by the nonlinear superpositions of KMBs, ABs, and PS [1].

In the real world, actual wave dynamics are often composed of a nonlinear superposition of some simple solutions, and several lower-order waves can be combined into a more complicated structure with higher intensity, which indicates that the novel characteristics of higher-order waves deserve special attention [1,8–15]. Recently, significant progress on the higher-order nonlinear waves has been achieved in both experimental and theoretical studies [1,8–15]. For example, Kibler et al. have reported a

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novel fiber-based test bed using tailored spectral shaping of an optical-frequency comb to excite the formation of two ABs that collide during propagation [10]. Super RWs with an amplitude of up to five times the background value have been observed in a water wave tank by Chabchoub et al. [11]. In the theoretic aspects, Kedziora et al. have systematically studied the breather solutions of the second-order NLS equation [1] and presented a systematic classification for higher-order RW solutions [9]. Compared with the first-order RW solutions, the higher-order ones with more stronger localization and higher intensities have such complex patterns as triangular, pentagram, hexagram, enneagram, hendecagrams structures and so on [9]. These RWs are not localized at the same position but arrayed spatiotemporally in elegant geometries [9]. He et al. have introduced a type of mechanism for generating higher-order RWs of the NLS equation, i.e., the progressive fusion and fission of n degenerate breathers associated with a critical eigenvalue λ_0 create the n-order RWs [12]. Guo et al. have discovered higher-order bright RW structures within the coupled NLS (CNLS) equations [13] and derivative NLS (DNLS) equation [14]. Chen et al. have also demonstrated the dark three-sister RWs in normally dispersive optical fibers with random birefringence in the frame of the CNLS equations [15]. Additionally, there are many other models with higher-order RWs such as the Gerjikov–Ivanov equation [16,17], modified NLS equation [18], Sasa–Satsuma equation [19–21], and NLS and Maxwell–Bloch equations [22,23], to name a few.

In addition to the breather (exponential functions) and RW solutions (rational functions), Kedziora et al. have found the semirational solution of the NLS equation [1]. Such type of solution can be regarded as the nonlinear superposition of an AB (or a KMB) with a PS [1]. Fabio Baronio et al. have reported that the CNLS equations admit the vector semirational solution which is a combination of rational and exponential functions and can be used to describe the interaction between PS and breather with nonconstant speed [24]. It should be pointed out that there have been some similar works published by Ling and Zhao during the same period [25,26]. The mixed rational-exponential solutions have also been discovered in the wave resonant-interaction models by Degasperis and Lombardo [27]. Furthermore, Qin and Chen have considered the higher-order vector semirational solutions for characterizing the evolutions of the n-order RW with multi-breather [28,29]. Noticing that if one of the plane-wave amplitudes approaches zero, the vector semirational solutions of the CNLS equations will degenerate the ones describing the interaction between the PS and soliton with nonconstant speed [24].

These investigations show that the semirational solution is one way to bridge the gap between a breather (or a soliton) and a RW [1,24]. In contrast with the standard soliton and breather solutions whose expressions are given in terms of exponentials, the semirational solution generically has a mixed rational and exponential expression [1,24], thus serves as a good candidate for studying the interactions between different nonlinear waves. One might ask: (1) Beyond the NLS equation and CNLS equations, do the semirational solutions exist in other models? (2) Do the higher-order semirational solutions have the determinant forms as soliton or breather ones and how to construct such representations? (3) In the regions of the breather interactions, how the higher-order RWs are formed by the smaller ones? Thus, continued investigation is required in order to better understand the higher-order breather and RW phenomena as well as their interactions.

In this paper, we focus on the DNLS equation [30]

$$i\,q_t - q_{xx} + i\,(q\,|q|^2)_x = 0, \tag{1}$$

which is an important model in such areas as plasma physics and nonlinear optics. Hereby, the subscript x and t stand for the partial derivatives with respect to x and t. In nonlinear optics, Eq. (1) governs the transmission of sub-picosecond pulse in single mode optical fibers [31]. In the context of plasmas, Eq. (1) not only describes the propagations of the small-amplitude Alfén waves in a low-β plasma [32], but also characterizes the evolutions of the large-amplitude magnetohydrodynamic waves in a high-β plasma [33]. By means of the Hirota method, inverse scattering transformation and Darboux transformation (DT), several types of exact solutions have been derived under the different boundary conditions including the one-soliton solution [34], two-soliton solution [35], n-soliton solution [36], stationary solutions [37] and breather solution [38]. Very recently, He [39] and Ling [14] have investigated the higher-order RW solutions as well as their dynamics of Eq. (1) via the modified DT (mDT). In this paper, our aim is to derive the higher-order semirational solutions in terms of the determinant forms for Eq. (1), and then we display different patterns of those solutions by manipulating the shift parameters. Additionally, we reveal the interaction mechanism between the RW and breather. To the best of our knowledge, such studies have not been reported elsewhere.

The outline of this paper is organized as follows: The determinant representation of semirational solutions for Eq. (1) will be derived through the modified Taylor expansion technique in Section 2; The patterns of semirational solutions, nonlinear wave interactions and formation mechanism of the higher-order RWs will be studied in Section 3; Our conclusions will be given in Section 4.

2. The semirational solution of Eq. (1)

First of all, let us review the DT of Eq. (1). The n-fold DT of Eq. (1) in terms of determinant form has obtained in Ref. [39], and the formulae for k-breather solution of Eq. (1) has been given as:

[39] Let $\Psi_j = (\phi_j,\ \varphi_j)^T$ $(j = 1, 2, \ldots, n = 2k)$ be distinct solutions related to the spectral parameters λ_j and seed solution $q^{[0]} = c\,e^{i(ax+bt)},\ b = a(-c^2 + a),\ a, c \in \mathbb{R}$, then $q_B^{[k]}$ given by the following formula is the breather solution of Eq. (1) as follows,

$$q_B{}^{[k]} = \frac{\Theta_{11}^2}{\Theta_{21}^2}q^{[0]} + 2i\frac{\Theta_{11}\Theta_{12}}{\Theta_{21}^2}, \tag{2}$$