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Experimental chaos detection in the Duffing oscillator

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ABSTRACT

This paper presents a comparative study of four algorithms namely the maximal Lyapunov exponent (MLE), 0–1 test, conditional entropy of ordinal patterns (CPE) and recently developed permutation largest slope entropy (PLSE) algorithm for experimental chaos detection in the Duffing oscillator. We consider an electrical model of the Duffing oscillator and its equivalent electronic circuit for generating the data to validate the effectiveness of the algorithms. The performance of the PLSE is compared with the 0–1 test and the CPE algorithms on the data set obtained from the simulated circuit; and with the MLE for the data collected from the experimental circuit. The experimental data are acquired using a digital oscilloscope with 1 MHz sampling frequency. From the comparison of the experimental spectra of the four methods with the analog phase portraits of the real system, it appears that the PLSE is the more reliable algorithm for chaos detection from experimental data.

1. Introduction

During the last three decades, a particular attention has been accorded to nonlinear systems exhibiting chaotic behaviors. Detecting chaos in real-world systems is difficult and remains a challenging task as the acquired time series data are always contaminated with noise and simulation models do not exactly approximate the behavior of the real-world systems [1–4]. So, it remains difficult for the results from theoretical models to match those from experimental data. Another difficulty encountered is to make the control parameters of the model to fit those of the real-world system. Indeed, it is not easy to model the adaptive properties of the real-world dynamical system parameters.

Thus, in order to detect chaos in a real-world system, the detection algorithm should be robust against noise. Many attempts based on the application of the Lyapunov exponent to real-world time series have been unsuccessfully undertaken, as the Lyapunov exponent is extremely sensitive to the noise [5]. Most of the existing algorithms use noise free data generated by simulation or directly from some particular digital circuits for computing the maximal Lyapunov exponent (MLE) [5,6]. Moreover, the algorithms used for computing the Lyapunov exponent are extremely time consuming, despite the proposed improvements

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[5]. Since then, many other algorithms have emerged and some of the most popular are the 0–1 test and those based on ordinal pattern analysis [7–13].

The 0–1 test is known to be robust against noise and therefore constitutes a good candidate for chaos detection from real-world data [7,14–16]. It directly applies to the time series and allows to detect the chaotic nature of the corresponding dynamics thanks to its binary chaos indicator, namely K: $K = 0$ for regular and $K = 1$ for non-regular dynamics. The test neither requires phase space reconstruction, nor the modeling equations of the underlying system. It has been successfully applied to continuous time systems, discrete time systems, integer-order systems as well as fractional-order systems. Despite the effectiveness of the 0–1 test, it is time consuming and cannot be used for real-time implementation [17].

In the group of ordinal patterns methods, the permutation entropy (PE) introduced by Bandt and Pompe is used in many applications due to its conceptual and computational simplicity [18]. The PE is based on the ordinal pattern analysis and is easily calculated for any type of time series. It has been also successfully applied to the study of structural changes in time series and the underlying system dynamics [8–12,19]. However, as there is no particular value or property of the PE for the characterization of regular dynamics, some improvements have been proposed [20,21], including the conditional entropy of ordinal patterns (CPE) [22]. The CPE was designed to provide more reliable estimation of the Kolmogorov–Sinai (KS) entropy than the PE, hence it is suitable to detect regular dynamics with zero entropy.

Recently, we proposed a new approach based on the entropy related to the largest slopes of the permutations [23] to address the limitations of the PE and related improvements to detect regular dynamics with zero entropy. We have shown that the permutation largest slope entropy (PLSE) is an effective approach for distinguishing between regular and non-regular dynamics and to detect the phase space period of stable limit cycles. This method achieves zero entropy from the observation time T much smaller than $n!$, where n is the permutation order. We have also shown that the use of permutation largest slopes allows to save computational time.

The effectiveness of the 0–1 test for chaos detection from experimental data has been studied in [24–26]. In the current work, we further undertake another experiment using a different system in order to confirm the effectiveness of detection and compare its performance with the ordinal pattern algorithms. As the 0–1 test is known to be time consuming, we further apply experimental data to the CPE and the PLSE which are known to be fast and easy to implement [13,22,23,27]. Such a comparison may be helpful in the choice of the nonlinear algorithm for the analysis of real-world time series.

Various electronic design of chaotic oscillators using discrete active devices and integrated circuit realizations are currently reported [28]. Some designers prefer field-programmable gate arrays (FPGA) for implementing nonlinear mathematical functions [29–31]. Although these realizations are consistent with their simulation models, it should be pointed out that they are using fixed point arithmetic with a limited precision for generating the data set . Our purpose is to consider an experimental system of a real-world dynamical system whose exact number of tuning parameters, their variation laws as well as interactions between the system components are not exactly defined. We also require a reality-based data acquisition process including sampling and quantization errors as well as the observational noise. We thus considered the Duffing oscillator, given that: it has been widely studied theoretically, various electrical models have been proposed and it is known to exhibit chaotic behaviors under certain forcing frequencies or voltages. We especially focus our study on the electrical non-autonomous model proposed by Leuciuc A. as it requires exclusively resistors and diodes for the implementation of the nonlinear term [32]. To illustrate the effectiveness of PLSE method, we analyze the data obtained from the experimental duffing oscillator circuit acquired using a digital storage oscilloscope sampled at 1MHz frequency. The paper presents a brief recall of the 0–1 test, the CPE and the PLSE, the modeling of the Duffing oscillator, and finally compares the simulation and experimental results for the four algorithms.

2. Brief recall of the detection methods

2.1. The binary 0–1 test

The 0–1 test is a binary test for chaos detection which does not require any prior knowledge on the equations of the underlying systems. The test has been successfully applied to maps and flows. In this section we briefly review how the test is implemented for continuous time systems. More details about implementation of the 0–1 test can be found in [14].

In an experimental data, $\phi(j)$ is a discrete measurement data in time index j for $j = 1, 2, \ldots$ then define the translational variables $p_{c\tau_s}$ and $q_{c\tau_s}$ as

$$p_{c\tau_s}(n) = \sum_{j=1}^{n} \phi(j) \cos jc \tag{1}$$

and

$$q_{c\tau_s}(n) = \sum_{j=1}^{n} \phi(j) \sin jc \tag{2}$$

where c refers to the frequency that depends on the sampling period and $n = 1, \ldots n_{cut}$ with $n_{cut} = N/10$.