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Review

Spontaneous formation of bright solitons in self-localized impurities in Bose–Einstein condensates

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We study the formation of bright solitons in the impurity component of Bose–Einstein condensate–impurity mixture by using the time-dependent Hartree–Fock–Bogoliubov theory. While we assume the boson–boson and impurity–boson interactions to be effectively repulsive, their character can be changed spontaneously from repulsive to attractive in the presence of strong anomalous correlations. In such a regime the impurity component becomes a system of effectively attractive atoms leading automatically to the generation of bright solitons. We find that this soliton decays at higher temperatures due to the dissipation induced by the impurity–host and host–host interactions. We show that after a sudden increase of the impurity–boson strength a train of bright solitons is produced and this can be interpreted in terms of the modulational instability of the time-dependent impurity wave function.

1. Introduction

A soliton is a self-focusing solitary wave that maintains its shape while it travels at constant speed and arises from a balance between nonlinear and dispersive effects. Bose–Einstein condensates (BECs) constitute a best environment for studying nonlinear macroscopic excitations in quantum systems. Excitations in the form of dark solitons with repulsive interaction have been realized in [1–3] one and half decade ago.

Bright solitons have been observed in BECs of ^{7}Li in quasi-one-dimensional (1D) regime [4,5]. The observation of bright solitons was therefore possible only by means of the Feshbach resonance and then tuning of the interactions from repulsive to attractive during the experiments. In the experiment of Strecker et al. [5], the formation of the bright soliton trains has been interpreted as due to quantum mechanical phase fluctuations of the bosonic field operator [6]. Bright soliton trains can be also generated in a BEC embedded in a quantum degenerate Fermi gas [7] as a result of a competition between two interparticle interactions: boson–boson collisions which are effectively repulsive and boson–fermion collisions which are attractive.

In this paper we propose a novel scheme to realize bright solitons in quasi-1D atomic quantum gases. In particular, we study the formation of bright solitons in the impurity component of BEC–impurity mixture at finite temperature by employing a versatile model known as time dependent Hartree–Fock–Bogoliubov (TDHFB) [26,27]. Experimentally, such mixtures have been already realized with a medium composed of either bosonic [8–11] or fermionic atoms [12–14]. Impurities in a Bose gas (Bose polarons) have been the subject of intense theoretical [15–25] studies. An important feature of these mixtures is that when neutral impurity atoms immersed in a BEC can spontaneously form a self-localize state. This localized state, within the strong

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coupling approach, exhibits a solitonic behavior at both zero and finite temperatures [17,26] in quasi-1D geometry. These solitons are reminiscent of the well known optical wave solitons [28].

Here, we have pointed out that the bright solitons can be created spontaneously in the impurity component for high anomalous density. This latter quantifies the correlations between pairs of condensed atoms with pairs of noncondensed atoms and rises with interactions strength. It was shown that the anomalous correlations play a crucial role on the stability of BEC and on the occurrence of the superfluidity [33]. We find that the single bright soliton decays at nonzero temperatures, with the decay rate increasing with rising temperature owing to the host–host and impurity–host interactions. In addition, we show that bright soliton trains can be produced automatically due to the modulational instability (MI) of the evolving classical phase in the impurity component of a harmonically trapped BEC–impurity mixture even with repulsive impurity–boson and boson–boson interactions. These trains generate without changing the trap geometry as has been suggested in [7], neither using the Feshbach resonance as it has been observed in [4,5], or even without imprinting the initial wave function with a fluctuating phase as is shown in [6]. The MI pattern associated with the attractive interaction plays a key role in the formation of bright solitons in a pure BEC [29,30]. By investigating the time evolution of soliton trains, we find that the number of bright solitons is increased with increasing the impurity–boson interactions.

This paper is organized as follows. In Section 2, we briefly review the main features of our theoretical approach. Section 3 is dedicated to analyze the behavior of a single soliton in the impurity component of a BEC–impurity mixture where we solve analytically and numerically the generalized self-focussing nonlinear Schrödinger equation. Section 4 presents the generation strategy of bright soliton trains in a trapped BEC–impurity mixture. We show in particular how the solitary wave formation occurs in the impurity through the MI. Section 5 is devoted to conclusion.

2. Formalism

We consider a mobile impurity of mass m_I immersed in a BEC of atoms of mass m_B at finite temperature. The impurity–boson interaction and boson–boson interactions have been approximated by the contact potentials $g_B\delta(\mathbf{r}-\mathbf{r}')$ and $g_{IB}\delta(\mathbf{r}-\mathbf{r}')$, respectively. We neglect the mutual interactions of impurity atoms since we assume that their number and local density remains sufficiently small [15,16] and hence there is no impurity fluctuation. The TDHFB equations which govern the dynamics of the condensate, the thermal cloud, the anomalous density and the impurity read [26,27]

$$i\hbar\dot{\Phi}_B=\left[h_B^{sp}+g_B((\beta-2)n_B+2n+\gamma n_I)\right]\Phi_B, \tag{1a}$$

$$i\hbar\dot{\tilde{m}}=4\left[h_B^{sp}+g_B(2G\tilde{m}+2n+\gamma n_I)\right]\tilde{m}, \tag{1b}$$

$$i\hbar\dot{\Phi}_I=\left[h_I^{sp}+g_{IB}(n_B+\tilde{n})\right]\Phi_I. \tag{1c}$$

In the set (1), $h_B^{sp}=-(\hbar^2/2m_B)\Delta+V_B$ and $h_I^{sp}=-(\hbar^2/2m_I)\Delta+V_I$ are, respectively the single particle Hamiltonian for the condensate and the impurity, with V_B and V_I being, respectively the condensate and the impurity trap potentials. $\Phi_B(\mathbf{r})=\langle\hat{\psi}_B(\mathbf{r})\rangle$ is the condensate wave function, $n_B(\mathbf{r})=|\Phi_B(\mathbf{r})|^2$ is the condensed density, $\Phi_I(\mathbf{r})=\langle\hat{\psi}_I(\mathbf{r})\rangle$ is the impurity wave function, $n_I(\mathbf{r})=|\Phi_I(\mathbf{r})|^2$ is the density of impurity atoms, the noncondensed density $\tilde{n}(\mathbf{r})$ and the anomalous density $\tilde{m}(\mathbf{r})$ are identified respectively with $\langle\hat{\psi}^+(\mathbf{r})\hat{\psi}(\mathbf{r})\rangle-\Phi_B^*(\mathbf{r})\Phi_B(\mathbf{r})$ and $\langle\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r})\rangle-\Phi_B(\mathbf{r})\Phi_B(\mathbf{r})$, where $\hat{\psi}^+$ and $\hat{\psi}$ are the boson destruction and creation field operators, respectively. The total density in BEC is defined by $n=n_B+\tilde{n}$. The dimensionless parameters $\beta=U/g_B$ with $U=g_B(1+\tilde{m}/\Phi_B^2)$ being the renormalized coupling constant [26,27], $G=\beta/4(\beta-1)$ and $\gamma=g_{IB}/g_B$ is the relative coupling strength. For $\beta=1$, i.e. $\tilde{m}/\Phi_B^2=0$, Eq. (1a) reduces to the HFB–Popov equation which is safe from all ultraviolet and infrared divergences and thus provides a gapless spectrum. For $0<\beta<1$, G is negative and hence, $\tilde{m}$ has a negative sign. For $\beta>1$, G is positive, and thus, $\tilde{m}$ becomes a positive quantity.

Neglecting the mean-field interaction energy between bosons and impurity components i.e. $g_{IB}=0$ and setting $\tilde{n}=\tilde{m}=0$, one recovers the well known Gross–Pitaevskii equation describing a degenerate Bose gas at zero temperature and the Schrödinger equation describing a noninteracting impurity system. For further computational details, see Refs.[26,27,31–34].

In our formalism the noncondensed and the anomalous densities are not independent. By deriving an explicit relationship between them, it is possible to eliminate $\tilde{n}$ via [26,27]:

$$I=(2\tilde{n}+1)^2-4|\tilde{m}|^2. \tag{2}$$

One can easily check by direct substitution that once Eq. (2) holds initially, it remains true during the dynamical evolution. The simplified set of equations are then the coupled Eq. (1) with $\tilde{n}$ is replaced by the expression (2). In the uniform case, by working in the momentum space, $I_k=\coth^2(\varepsilon_k/T)$ [33], where ε_k is the excitation energy of BEC. At zero temperature $I=1$ [27], and hence, Eq. (2) reduces to $\tilde{n}(\tilde{n}+1)=|\tilde{m}|^2$. Therefore, the expression of I clearly shows that $\tilde{m}$ is larger than $\tilde{n}$ at low temperature, so the omission of the anomalous density in this situation is principally unjustified approximation and wrong from the mathematical point of view. Importantly, the expression of I not only renders the set (1) close but also enables us to reduce the number of equation making the numerical simulation easier.

Moreover, what is important in the TDHFB approach for Bose systems is that there have been no assumptions on weak interactions. Therefore, the theory is valid even for strong interactions [31,32]. In addition, the TDHFB Eqs. (1) satisfy the total number of particles and the energy conservation law, and they provide a gapless spectrum [27].