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Large Amplitude Flexural Vibration of the Orthotropic Composite Plate Embedded with Shape Memory Alloy Fibers

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Abstract

The free and forced vibration of large deformation composite plate embedded with shape memory alloy (SMA) fibers is investigated. A thermo-mechanical constitutive equation of SMA proposed by Brinson et al. is employed and the constitutive equations for evaluation of the properties of a hybrid SMA composite laminate are obtained. Based on the nonlinear theory of symmetrically laminated anisotropic plates, the governing equations of flexural vibration in terms of displacement and stress functions are derived. The Galerkin method has been used to convert the original partial differential equation into a nonlinear ordinary differential equation, which is then solved with harmonic balance method. The numerical results show that the relationship between nonlinear natural frequency ratio and temperature for the nonlinear plate has similar characteristics compared with that of the linear one, and the effects of temperature on forced response behavior during phase transformation from Martensite to Austenite are significant. The effects of the volume fraction of the SMA fiber, aspect ratio and free vibration amplitude on the dynamical behavior of the plate are also discussed.

Keywords: smart materials; large-deflection plate; composite; nonlinear vibration

1 Introduction

In recent ten years, great progress has been made on the research of shape memory alloy (SMA) reinforced smart structure systems. Through the design of SMA intelligent composite structure systems with highly integrated sensors and actuators, the control of complex flexible structures with rigorous weight limitations such as aeronautic structures has become possible. For example, the buckling and vibration characteristics of SMA reinforced composite plate and beam structures have been greatly improved when compared to traditional composite structures, this makes SMA be widely used in buckling and vibration control[1-2].

For the structures with high strength and large flexibility used in aircraft design, the geometric nonlinear effect resulted from large deformation cannot be ignored. For instance, the shapes of the panels of the supersonic aircraft will be changed due to the large thermal deflections induced by aerodynamic heating, which will affect the aerodynamic characteristics and reduce the flight performance of the aircraft. In order to control such kind of nonlinear structures, it is necessary to know the effect of geometric nonlinearity on the vibration of structures. However, most of the researches on SMA reinforced composite plates and beams are based on the classic small deflection theory and only a few are concerning with the large deflection nonlinearity issues.

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Based on von Karman plate theory, Chu L C[3] established the nonlinear finite element model for the SMA fiber reinforced composite large-deflection plate and studied the suppression of the flutter of panels of aircraft. Zou Jing[4] derived the incremental finite element motion equation for the nonlinear composite laminate embedded with SMA fibers on the basis of the virtual work principle, and the bending, thermo-buckling and post-buckling issues of the SMA reinforced composite laminate under transverse loading were discussed. Dano M L et al.[5] studied the effect of SMA on the snap-through characteristics of the nonlinear unsymmetric fiber-reinforced composite laminate with large-deflections. They established the approximate theory to analyze the snap-through characteristics of the unsymmetric fiber-reinforced composite laminate activated by SMA wires, where the mechanical properties of the laminate are predicted with the assumed strain-displacement field, Rayleigh-Ritz method and the virtual work principle, and the law of snap-through varying with the temperature was obtained through solving relevant simultaneous equations and the equation describing SMA properties. Park J S et al.[6-7] established the finite element model of composite laminates and supersonic plates under nonlinear vibration and nonlinear flutter by using the von Karman plate theory, the one-order shearing deformation plate theory and the first-order piston theory. The boundary problems of thermo-post-buckling nonlinear vibration and flutter were then investigated for the SMA fiber reinforced composite plate under thermal and aerodynamic loading. Cho M et al.[8] studied the deformation of the nonlinear composite plate with two-way shape memory effect with the theory of the one-order shearing deformation plate with large deflections and the thermo-mechanical constitutive equation proposed by Lagoudas et al.

Based on the above literature review, we realize that: ① most of the current researches on the geometric nonlinearity of the SMA fiber (or layer) reinforced composite plate are limited to using the finite element numerical method[3-7], and little work has been reported to use the nonlinear elastic theory to get the analytical vibration solution for the SMA reinforced nonlinear anisotropic laminates; ② about the description of the mechanical behavior of SMA, some are based on the expression of the approximate experimental fitting[3], the others use the approximate data derived from the curves of SMA[6-7], and the well-developed and practical constitutive equation of SMA based on the phenomenal theory such as Brinson's equation[4,9] is hardly used; ③ the effect of SMA fibers on the nonlinear vibration of composite plates still needs to be investigated further.

In this paper, the free and forced vibration of the SMA reinforced composite laminates with large deflections will be studied. The thermo-mechanical constitutive equation of SMA proposed by Brinson et al.[4] and the mixture theory for evaluating the properties of laminates are employed to establish the constitutive equation of the SMA reinforced composite laminates with large deflections. Based on the nonlinear theory of symmetrically laminated anisotropic plates, the transverse vibration equation and the compatible equation will be derived in terms of the transverse deflection and the stress function. The Galerkin approximate method and the harmonic balance method (HBM) will be used to solve the Duffing's differential equation and to study the effect of the content of SMA fibers, the nonlinear vibration amplitude and the excitation force amplitude on the natural frequency and steady-state response of the system.

2 Basic Theories

2.1 The constitutive equation of the anisotropic laminate with embedded SMA fibers

As the SMA reinforced anisotropic lamina has the following off-axis stress-strain relation[10]

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\bar{\boldsymbol{Q}}_{ij}^{(k)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}-\tilde{\boldsymbol{Q}}_{ij}^{(k)}\begin{Bmatrix}\alpha_x^*\\ \alpha_y^*\\ \alpha_{xy}^*\end{Bmatrix}\Delta T+\begin{Bmatrix}\sigma_{\mathrm{rx}}^*\\ \sigma_{\mathrm{ry}}^*\\ \tau_{\mathrm{rxy}}^*\end{Bmatrix} \quad (1)$$