Commun Nonlinear Sci Numer Simulat 20 (2015) 74–78

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Group analysis of the drift–diffusion model for quantum semiconductors

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ARTICLE INFO

Article history:
Received 4 May 2014
Accepted 19 May 2014
Available online 24 May 2014

Keywords:
Quantum semiconductor
Quantum drift–diffusion model
Group analysis
Nonlinear self-adjointness
Conservation laws
Exact solutions

ABSTRACT

In the present paper a quantum drift–diffusion model describing semi-conductor devices is considered. New conservation laws for the model are computed and used to construct exact solutions.

1. Introduction

Ultra small semiconductor devices can be found in thousands of products such as computers, cell phones and other modern appliances. Mathematical models of such devices naturally involve quantum mechanical considerations. See, e.g., [1–3] and the references therein. Accordingly, these models are described by quite complicated systems of nonlinear partial differential equations. Among these models macroscopic quantum models are of special interest because they are easier to analyse than the microscopic models. The simplest one is the so-called quantum drift–diffusion model, obtained by assuming that the relaxation time of the semiconductor device is very small, involves such microscopic quantities as electron density n and electrostatic potential V. The model is given by the following system of two partial differential equations:

$$
\begin{aligned}
&n_t = \operatorname{div}\left[\varepsilon^2 n \nabla\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right) + \theta\nabla(n) + n\nabla V\right],\\
&\lambda^2 \Delta V = C(x) - n.
\end{aligned}
\tag{1}
$$

where $C(x)$ describes the distribution of background ions, $\varepsilon > 0,\ \lambda > 0$ and θ are physical constants.

To the best of our knowledge, no exact solutions of the system (1) have been presented in the literature. Our aim is to fill this gap and to construct exact solutions of the system (1) using the recent method of conservation laws [4]. The first equation of the system (1) has the conservation form. However application of the method of conservation laws to this conserva-

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http://dx.doi.org/10.1016/j.cnsns.2014.05.020
1007-5704/

tion law gives only the stationary solution. Therefore we are interested in the question wether there are other conservation laws.

For the sake of simplicity we will consider the one-dimensional case. In this case Eq. (1) is written:

$$\begin{aligned} n_t &= \varepsilon^2\left[-\frac{1}{2}n_{xxxx}+\frac{1}{n}\left(n_x n_{xxx}+n_{xx}^2\right)-\frac{5n_x^2 n_{xx}}{2n^2}+\frac{n_x^4}{n^3}\right]+\theta n_{xx}+nV_{xx}+n_x V_x, \\ \lambda^2 V_x &= C(x)-n. \end{aligned} \tag{2}$$

We will construct conservation laws to the system (2) by the method of nonlinear self-adjointness [5].

2. Nonlinear self-adjointness

Let us write the system (2) in the form

$$F_1 = 0, \quad F_2 = 0, \tag{3}$$

where

$$\begin{aligned} F_1 &= -n_t+\varepsilon^2\left[-\frac{1}{2}n_{xxxx}+\frac{1}{n}\left(n_x n_{xxx}+n_{xx}^2\right)-\frac{5n_x^2 n_{xx}}{2n^2}+\frac{n_x^4}{n^3}\right]+\theta n_{xx}+nV_{xx}+n_x V_x, \\ F_2 &= -\lambda^2 V_x + C(x) - n. \end{aligned} \tag{4}$$

By definition [5] the adjoint system to Eq. (3) is written

$$F_1^* = 0, \quad F_2^* = 0, \tag{5}$$

where

$$F_1^* = \frac{\delta\mathcal{L}}{\delta n}, \quad F_2^* = \frac{\delta\mathcal{L}}{\delta V}, \tag{6}$$

with

$$\mathcal{L} = v^1 F_1 = v^2 F_2.$$

Here $v^1,\ v^2$ are new dependent variables. The calculation gives

$$\begin{aligned} F_1^* &= \varepsilon^2 v^1_{xxxx} + 2\varepsilon^2\frac{n_x}{n}v^1_{xxx} + \left(2\varepsilon^2\frac{n_{xx}}{n} - \varepsilon^2\frac{n_x^2}{n^2} - 2\theta\right)v^1_{xx} + 2V_x v^1_x - 2v^1_t + v^2, \\ F_2^* &= n v^1_{xx} + n_x v^1_x + \lambda^2 v^2_x, \end{aligned} \tag{7}$$

According to [5] the system (3) will be nonlinearly self-adjoint if there exists a substitution

$$v^1 = \varphi_1(t,x,n,V), \quad v^2 = \varphi_2(t,x,n,V), \tag{8}$$

such that the following equations are satisfied:

$$F_1^*|_{(8)} = \mu_1^1 F_1 + \mu_1^2 F_2, \quad F_2^*|_{(8)} = \mu_2^1 F_1 + \mu_2^2 F_2, \tag{9}$$

where μ_β^α are undetermined coefficients. The notation $F_1^*|_{(8)},\ F_2^*|_{(8)}$ means that the variables $v^1,\ v^2$ and their derivatives in (6) are eliminated by means of the substitution (8).

Solving Eq. (9), we obtain the following substitution:

$$v^1 = \Phi(t), \quad v^2 = \Phi'(t), \tag{10}$$

where $\Phi(t)$ is an arbitrary function and $\Phi'(t)$ is its derivative.

3. Symmetries and conservation laws

The system (2) has two symmetries:

$$X_1 = \frac{\partial}{\partial t}, \quad X_2 = g(t)\frac{\partial}{\partial V}, \tag{11}$$

where $g(t)$ is an arbitrary function.

The conserved vector associated with a symmetry

$$X = \xi^i\frac{\partial}{\partial x^i} + \eta^\alpha\frac{\partial}{\partial u^\alpha},$$