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An exact solution for the 3D MHD stagnation-point flow of a micropolar fluid

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ABSTRACT

The influence of a non-uniform external magnetic field on the steady three dimensional stagnation-point flow of a micropolar fluid over a rigid uncharged dielectric at rest is studied. The total magnetic field is parallel to the velocity at infinity. It is proved that this flow is possible only in the axisymmetric case. The governing nonlinear partial differential equations are reduced to a system of ordinary differential equations by a similarity transformation, before being solved numerically. The effects of the governing parameters on the fluid flow and on the magnetic field are illustrated graphically and discussed.

1. Introduction

The recent industrial processes are characterized by the use of new materials which cannot be described by Newtonian fluids. Due to this reason, many non-Newtonian models have been proposed. Among these models, the micropolar fluids have been introduced by Eringen [1] in order to take into consideration the effects of local structure and micro-motions of the fluid particles which cannot be described by the classical models. The incompressible micropolar fluids represent liquids consisting of rigid, randomly oriented spherical particles suspended in a viscous medium, where the deformation of fluid particles is ignored. The related mathematical model is based on the introduction of a new vector field (the microrotation) which describes the total angular velocity of the particles rotation. Hence a new equation is added representing the principle of conservation of local angular momentum. Micropolar fluids describe the behavior of polymeric fluids, exotic lubricants, biological liquids, microemulsions, alloys, colloidal suspensions, polymeric blends and liquid crystals so that they have many applications in the chemical, pharmaceutical, engineering and food industries. In parallel with practical applications [2–4], the theoretical aspects of the solution have been investigated by many authors [5–10].

A very vast amount of research on the effects of an electromagnetic field on the micropolar fluid flow under different conditions and in the presence of various physical effects has been reported [11–18,9,19]. These efforts have been made to study the MHD problems by many physicists and mathematicians due to their relevant applications, complexity and mathematical challenges. In particular, a relevant physical situation studied by several authors ([20–22] and the references quoted herein) is when the flow and the magnetic field are aligned at infinity.

An important example of the mutual interaction between the fluid flow and the electromagnetic field is the MHD stagnation-point flow. The orthogonal two-dimensional stagnation-point flow of a Newtonian fluid on a flat plate first studied by Hiemenz [23] was extended to the three-dimensional case by Homman [24]. From the mathematical point of view,

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stagnation-point flow is an important exact solution of the Navier–Stokes equations which belongs to the similarity solutions class. By similarity transformations, the PDEs which govern the motion are reduced to a system of ODEs. Similarity solutions describe fundamental physically relevant problems and are used as test for the accuracy of numerical methods. The stagnation-point flow describes physically a jet of fluid which impinges on a rigid body. The problem of stagnation-point flow was extended in numerous ways to include various physical effects [25–28]. In particular, as far as the micropolar fluids are concerned, the plane stagnation-point flow was studied in [29], while the three-dimensional one in [30]. Previously Ahmadi [31] obtained self-similar solutions of the boundary layer equations for micropolar flow imposing a condition on the material parameters which make the equations to contain only one parameter. This approach has been followed by several Authors (see for example [32–34,4]). We point out that in our research we have not required any condition so that three material parameters appear in the dimensionless ODEs.

In this paper we study the influence of a non-uniform external magnetic field on the steady three dimensional stagnation-point flow of a micropolar fluid. The only published result about this physical situation can be found in [35] for the Newtonian fluids.

We examine the 3D stagnation-point flow of a micropolar fluid filling the half-space $x_2 \geqslant 0$ when the total magnetic field $\mathbf{H}$ is parallel to the velocity at infinity. We search $\mathbf{H}$ depending on two sufficiently regular unknown functions. The whole space is permeated by a non-uniform external magnetic field $\mathbf{H}_e$ while the external electric field is absent. The expression of $\mathbf{H}_e$ assures that if we consider the 3D stagnation point flow of an inviscid fluid (see [35]) we have that $\mathbf{H}_e$ coincides with the total magnetic field and it is parallel to the fluid velocity in all the half space $x_2 > 0$. Due to the no-slip condition for the velocity of the micropolar fluid at $x_2 = 0$, this alignment is disrupted near the boundary. However, as it is reasonable from the physical point of view, the viscosity effects occur only in a boundary layer and so we require that the total magnetic field and the fluid velocity are parallel at infinity. It is expected that this request is satisfied if the external magnetic field is sufficiently weak.

The region occupied by the fluid is bordered by the boundary of a solid obstacle which is a rigid uncharged dielectric at rest. We underline that many Authors ignore the details of the electromagnetic field in the solid region but the relevance of the problem to any physical situation may be in doubt if we do not join the solution in the fluid to a suitable solution in the solid. In [35] it is proved that the expression of the electromagnetic field in the solid is formally the same independently of the fluid model over the solid.

In the first section we recall the results obtained in [35] when the fluid over the solid is inviscid. The analysis of the inviscid case is very important because, as it is reasonable from the physical point of view, the viscosity occurs only near the boundary. So we assume that at infinity the flow of the micropolar fluid approaches the flow of an inviscid fluid for which the stagnation-point is shifted from the origin.

The goal of this paper is to prove that such steady 3D MHD stagnation-point flow of a micropolar fluid is possible only if the flow is axisymmetric. The study of this problem leads to a non linear ordinary differential problem which depends on three material parameters describing the micropolar nature (c_1, c_2, c_3) and on two parameters R_m (Reynolds number or magnetic Prandtl number) and β_m (Alfvén number) related to the magnetic nature of the flow. By solving numerically the problem, we find that, as usual in stagnation-point flows, the influence of the viscosity appears only in layer lying the boundary whose thickness depends on R_m and β_m. More precisely, it increases as β_m increases, while it decreases as R_m increases.

Some numerical examples and pictures are given in order to illustrate the effects due to the magnetic field on the behavior of the solution. The numerical results are obtained by using the MATLAB routine bvp4c, which is described in [36].

2. Preliminaries

Let us consider the steady three-dimensional MHD flow of a homogeneous, incompressible, electrically conducting micropolar fluid near a stagnation-point filling the half-space $\mathcal{S}$ (see Fig. 1), given by

$$\mathcal{S} = \left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 : (x_1, x_3) \in \mathbb{R}^2, x_2 > 0 \right\}. \tag{1}$$

The coordinate axes are chosen in order to have that the stagnation-point coincides with the origin and the canonical base of $\mathbb{R}^3$ is denoted by $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$. $\partial\mathcal{S}$, i.e. the plane $x_2 = 0$, is the boundary of a solid which is a rigid uncharged dielectric at rest occupying

$$\mathcal{S}^- = \left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 : (x_1, x_3) \in \mathbb{R}^2, x_2 < 0 \right\}. \tag{2}$$

In the absence of free electric charges and external mechanical body forces and body couples, the MHD equations for such a fluid are (see [6])

$$\begin{aligned}
&\mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + (\nu + \nu_r) \triangle \mathbf{v} + 2\nu_r (\nabla \times \mathbf{w}) + \frac{\mu_e}{\rho} (\nabla \times \mathbf{H}) \times \mathbf{H}, \\
&\nabla \cdot \mathbf{v} = 0, \\
&I \mathbf{v} \cdot \nabla \mathbf{w} = \lambda \triangle \mathbf{w} + \lambda_0 \nabla (\nabla \cdot \mathbf{w}) - 4\nu_r \mathbf{w} + 2\nu_r (\nabla \times \mathbf{v}), \\
&\nabla \times \mathbf{H} = \sigma_e (\mathbf{E} + \mu_e \mathbf{v} \times \mathbf{H}), \\
&\nabla \times \mathbf{E} = \mathbf{0}, \quad \nabla \cdot \mathbf{E} = 0, \quad \nabla \cdot \mathbf{H} = 0, \quad \text{in } \mathcal{S},
\end{aligned} \tag{3}$$