Commun Nonlinear Sci Numer Simulat 20 (2015) 159–173

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Thermodynamics of a bouncer model: A simplified one-dimensional gas

Edson D. Leonel [a,b,*], André L.P. Livorati [c,d]

[a] UNESP – Univ Estadual Paulista, Departamento de Física, Av.24A 1515, Bela Vista, 13506-900 Rio Claro, SP, Brazil
[b] The Abdus Salam – ICTP, Strada Costiera, 11, 34151 Trieste, Italy
[c] Instituto de Física, Univ São Paulo, Rua do Matão, Cidade Universitária, 05314-970 São Paulo, SP, Brazil
[d] School of Mathematics, University of Bristol, Bristol BS8 1TW, United Kingdom

ARTICLE INFO

Article history:
Received 6 December 2013
Received in revised form 8 May 2014
Accepted 8 May 2014
Available online 29 May 2014

Keywords:
Bouncer model
Diffusion in energy
Scaling

ABSTRACT

Some dynamical properties of non interacting particles in a bouncer model are described. They move under gravity experiencing collisions with a moving platform. The evolution to steady state is described in two cases for dissipative dynamics with inelastic collisions: (i) for large initial energy; (ii) for low initial energy. For (i) we prove an exponential decay while for (ii) a power law marked by a changeover to the steady state is observed. A relation for collisions and time is obtained and allows us to write relevant observables as temperature and entropy as function of either number of collisions and time.

1. Introduction

Modelling a dynamical system has become one of the most challenging subjects among scientists including physicists and mathematicians over years [1,2]. The modelling helps to understand in many cases how does the system evolves in time [3] was well as its description in parameter space [4,5] and whether it has or not a steady state [6]. Often the investigation leads to nonlinear dynamics [7] where complex structures can be observed in the phase space [8]. For conservative systems, the phase space may be classified under three different classes namely: (i) regular [9] where only periodic and quasi periodic orbits are present; (ii) mixed [10] whose phase space exhibits a coexistence of period, quasi periodic and chaotic behaviour and; (iii) ergodic [11] where only unstable and therefore unpredictable orbits are observed. For dissipative systems [12] the structure of the phase space commonly has attractors [13] that can be periodic [14] or chaotic [15].

In the large majority of the cases, a dynamical system is mostly governed by a set of differential equations. Quite often too they are coupled to each other. However, depending on the conserved quantities and symmetries, solutions of differential equations can be qualitatively (and many times quantitatively too) transformed into an application described by nonlinear mappings [16]. The mappings are characterised by discrete time evolution and have also a set of control parameters. Indeed they can control either the nonlinearity [17] as well as the dissipation itself [18].

The variation of the control parameters may lead quite often to the so called phase transitions [19,20]. In statistical mechanics, phase transitions are linked to abrupt changes in spatial structure of the system [21,22] and mainly due to variations of control parameters. In a dynamical system however, a phase transition is particularly related to modifications in the

* Corresponding author at: UNESP – Univ Estadual Paulista, Departamento de Física, Av.24A 1515, Bela Vista, 13506-900 Rio Claro, SP, Brazil. Tel.: +55 19 3526 9174; fax: +55 19 3526 9181.
E-mail address: edleonel@rc.unesp.br (E.D. Leonel).

http://dx.doi.org/10.1016/j.cnsns.2014.05.023
1007-5704/

structure of the phase space of the system [23,24]. Therefore near a phase transition, the dynamics of the system is described by the use of a scaling function [25,26] where critical exponents characterise the dynamics near the criticality.

In this paper we revisit a bouncer model [27,28] particularly focused on the description of some of its thermodynamical properties. The system consists of a classical particle, or in the same way an ensemble of non interacting particles, moving under the action of a constant gravitational field and suffering collisions with a moving platform. We are seeking then to understand and describe how does the system goes to the steady state for long enough time and how the control parameters influence the way the system goes. For the conservative case and depending on the control parameters [29], the system exhibits unlimited diffusion in energy [30] which is called as Fermi acceleration [31]. If the particles are considered as a sufficiently light, an ensemble of them may constitute an ideal gas. Therefore the unlimited diffusion in energy is in contrast with what is observed in day life. If we consider the moving wall as produced by atomic oscillations in a solid due to thermal heating with a constant external temperature T_e, a gas in a room does not absorbs infinite energy leading it to have unbound growth of temperature. Therefore Fermi acceleration is a phenomenon that cannot occur in gases and most probably is due to the fact that dissipation is present. Because the gas is of low density and particles are non interacting with each other, we consider the dissipation is due to inelastic collisions of the particles with the moving wall, but the particles do not interact between themselves. When interactions among the particles are considered, the system can be described as a granular material [32] allowing physical observables to be characterised [33,34] either in the presence [35] or absence [36] of gravitational field. Such approach is not considered in this paper given we are considering non interaction particles with low density. In model considered in this paper we introduce a dissipation parameter, more specifically a restitution coefficient, and describe the evolution of the system. Then the introduction of inelastic collisions of the particle with the wall suppresses the diffusion in energy. The evolution towards the stationary state for long enough time is described in two limits: (i) if the initial energy of the gas is sufficiently large and; (ii) if it is sufficiently small. For case (i) we prove an exponential decay is happening while for (ii) a power law marked by a changeover to the steady state is observed. We obtain so far a relation of the number of collisions and time and write the relevant observables like squared velocity, temperature of the gas and entropy as a function of either number of collisions and time. The system is show to be scaling invariant with respect to the control parameters and we found analytically the critical exponents describing a homogeneous generalised function. At the end we present some comparisons of the results obtained with typical values of known atomic oscillations as well as frequency of oscillation at a given temperature. Our results indicate the inelastic collisions are responsible for suppressing the energy's unlimited diffusion of the gas. An estimation of a restitution coefficient is given for Hydrogen molecule colliding with a solid made of copper.

This paper is organised as follows. In Section 2 we describe the model, give the expressions of the conservative and dissipative maps. Unlimited diffusion for energy is shown also for the conservative case. The stationary state and the evolution towards it is given here as a theoretical prediction. Section 3 is devoted to discuss the numerical results as well as the scaling properties. The critical exponents are obtained from either theoretical point of view as well as from numerical simulations. A scaling invariance for the ensemble of particles in a gas is confirmed by an overlap of different curves of average velocity onto a single and universal plot, after properly rescaling of the axis. As the system is constructed and described in terms of the number of collisions, Section 4 deals specifically with a connection of number of collisions and time. The latter being in principle easier to be measured in a real experiment. Section 5 discusses the connections with the thermodynamics finding particularly the expressions of the temperature of the gas, squared velocity as well as an expression for the entropy. Both as function of number of collisions as well as the time. Short discussion on the results are presented in Section 6 where, from our model, an estimation of the restitution coefficient for collisions of Hydrogen molecule is given. Conclusions are presented in Section 7.

2. The model and the map

The model we consider in this paper consists of a classical particle, or an ensemble of non interacting particles, moving in the presence of a constant gravitational field g suffering collisions with a time moving wall. The equation that describes the moving wall is given by

$$y_w(t) = \epsilon\cos(\omega t), \tag{1}$$

where ϵ denotes the amplitude of the moving wall while ω is the angular frequency. We suppose the gas of particles has a low density in the sense the particles are free to move with a constant mechanical energy without interacting with each other. They indeed exchange energy upon collisions with the moving wall. Depending on the phase of the moving wall the particles can gain or lose energy. It is assumed the motion of the particles is allowed only in the vertical direction, therefore making allusions to a simplified one-dimensional gas. Fig. 1 shows a schematic description of the model.

The dynamics of each individual particle, as usual in the literature [2], is described by a two-dimensional and nonlinear map for the variables velocity of the particle V and time t at each impact with the boundary. For a simplified gas model, the moving platform may be represented by a rigid wall or even the ground and the motion can be caused by the atomic oscillations at the edge of the wall. Because such atomic oscillations are too small [37] as compared to the positions or displacements made by each particle, we can approximate the description by assuming the time of flight of each particle is calculated