Commun Nonlinear Sci Numer Simulat 20 (2015) 186–198

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Detailed bifurcation analysis with a simplified model for advance heavy water reactor system

Vikas Pandey, Suneet Singh *

Department of Energy Science and Engineering, Indian Institute of Technology (IIT) Bombay, Powai, Mumbai, India

ARTICLE INFO

Article history:
Received 28 October 2013
Received in revised form 17 May 2014
Accepted 19 May 2014
Available online 24 May 2014

Keywords:
Floquet multiplier
Lyapunov coefficient
Limit point bifurcation of cycles
Generalized Hopf bifurcation

ABSTRACT

The bifurcation analysis of fixed points and limit cycles with a simplified mathematical model representing system dynamics of a boiling water reactor has been carried out, specifically parameter values for AHWR is used. The lumped parameter model that includes point reactor kinetics equation for neutron balance in the reactor core and one node model for fuel and coolant thermal hydraulics is used in the analysis. The nonlinearity due to reactivity is considered in the present model; while other nonlinearities due to heat transfer process between fuel–clad and fuel–coolant has been neglected. The system loses its stability via Hopf bifurcation as the system parameters are varied. The continuations of subcritical and supercritical Hopf points show the existence of limit point bifurcations of limit cycles (LPC). The codimension one and codimension two bifurcations of fixed points for the system have been analyzed. The stability of observed limit cycles has been analyzed by Floquet multiplier as well as by Lyapunov coefficient. The pattern of limit cycles and envelopes of limit cycles over the fixed points have been studied by numerical integrations and depicted by time history graphs.

1. Introduction

Nonlinear stability analysis of dynamical systems of nuclear reactors has been a very promising research area in the last few decades. Different mathematical models for nuclear reactors have been developed for nonlinear stability analysis with or without neutronics. Achard analytically investigated instabilities in the boiling channel without incorporating the neutronics in reactor system by quasi linear Hopf bifurcation analysis to predict the amplitude and frequency of limit cycles near the stability boundary in the unstable region [1]. Clause and Lahey used a homogeneous equilibrium two phase flow model without considering neutronics in a boiling system. The supercritical Hopf bifurcation was observed which leads to chaos via cascade of period doubling of limit cycles at low flow rate [2]. Rizwan-uddin and Dorning studied a heated boiling channel using two phase flow drift flux model without neutronics and found the occurrence of the supercritical Hopf bifurcation in the vicinity of marginal stability boundary [3]. Lee and Pan studied boiling channel with riser using same approach as Clausse and Lahey and found that the system leads to chaos via cascade of period doubling of limit cycles at high inlet subcooling [4].

A phenomenological model considering point reactor kinetics, one node representation of heat transfer process in fuel rods and thermal hydraulics of reactor channels have been developed and studied for nonlinear stability of a boiling water

* Corresponding author. Tel.: +91 22 2576 7843.
E-mail address: suneet.singh@iitb.ac.in (S. Singh).

http://dx.doi.org/10.1016/j.cnsns.2014.05.019

Nomenclature

C_0	steady state precursor concentration of delayed neutrons (m^{-3})
$C(\tau)$	precursor concentration of the delayed neutrons (m^{-3})
C_f	heat capacity of the fuel element in the reactor core ($J\,K^{-1}$)
h_{fg}	specific enthalpy of evaporation at saturation pressure ($J\,kg^{-1}$)
N_0	steady state neutron density (m^{-3})
$N(\tau)$	neutron density (m^{-3})
P_0	steady state reactor power (W)
$P(\tau)$	the reactor power (W)
$T_f(\tau)$	the average fuel temperature (K)
T_{f0}	steady state fuel temperature (K)
T_{sat}	saturation temperature of the coolant (K)
U	over all heat transfer coefficient including fuel conductivity ($W\,K^{-1}$)
v_g	specific volume of saturated vapor at saturation pressure
V_0	steady state void volume (m^3)
$V(\tau)$	void volume (m^3)
α_f	fuel temperature coefficient of reactivity (K^{-1})
α_v	void coefficient of reactivity (dimensionless)
β	delayed neutrons fraction (dimensionless)
Λ	neutron generation time (sec.)
λ_1	decay constant for the delayed neutron precursors (sec^{-1})
$\rho(\tau)$	reactivity multiplication factor
τ	time (sec.)
AHWR	advanced Heavy Water Reactor
BWR	boiling water reactor

reactor. It has been concluded that the limit cycles bifurcates via period doubling and lead to chaos [5,6]. The fifth order model of March-Leuba was reduced by Suzudo and Sinohara into third order model and studied analytically by bifurcation theory for nonlinear nuclear reactor dynamical system [7]. A lumped parameter model was studied by Lahey analytically for subcritical and supercritical Hopf bifurcation in the boiling channels [8]. The point reactor kinetics with different void reactivity feedback models have been proposed by Wang and Kondo and studied with central manifold method and bifurcation theory to find out the reason of excitation of limit cycles in the BWR dynamical system. It has been found that damping term in the void reactivity defines the types of limit cycles [9].

Karve et al. and Dokhane et al. developed a reduced order model which was analyzed by BIFDD code for bifurcation analysis [10–12]. The bifurcation analysis of nuclear reactor systems was done for the local Hopf bifurcation in these works. Rizwan-uddin further extended these studies using BIFDD for the bifurcation analysis of March-Leuba model and confirmed the existence of turning point bifurcation by numerical integrations [13]. However, it is observed that there are no studies on limit point bifurcation of limit cycles and generalized Hopf bifurcations in the context of nuclear reactor dynamical systems. In the present work, the codimension one and codimension two bifurcations of fixed points have been analyzed by observing eigenvalues and Lyapunov coefficients respectively. The codimension one bifurcation of limit cycles has been studied by observing Floquet multipliers. MATCONT, which is a mathematical package for numerical continuation, is used in the present study to detect the codimension one and codimension two bifurcations [14]. The mathematical model of nuclear reactor system used by Wahi and Kumawat has been adopted here for the analysis [15].

2. Mathematical background

The stability analysis of any physical system is study of change in temporal behavior of that system due to disturbances from its operating conditions or steady state conditions. Although, the linear stability analysis is easier to carry out and less computationally expensive, however, nonlinear stability of dynamical system is needed to give more characteristic information about the system. The qualitative and quantitative behavior of any physical system depends on the parameters of that particular system. The bifurcation analysis of any physical system, which is a part of nonlinear stability analysis, deals with analysis of non-uniqueness of solutions of system and variation in the multiplicity of solutions as the parameters of the system are varied. The Poincare–Andronov–Hopf bifurcation, generally known as Hopf bifurcation, has been observed by several authors for various mathematical models of nuclear reactors [16,17].

The stability of local fixed points of any nonlinear system can be analyzed from characteristic roots of the Jacobian matrix. The eigenvalues of the Jacobian matrix moves in the complex plane as system parameters are varied. The perturbation in the system decays and settles to a fixed point of the system, if real parts of all the eigenvalues of the Jacobian are negative and hence the system can be considered as stable. However, even if one eigenvalue has positive real part the system is unstable.