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Pinning impulsive synchronization of complex-variable dynamical network

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ABSTRACT

In this paper, pinning combining with impulsive control scheme is adopted to investigate the synchronization of complex-variable dynamical network. Based on the Lyapunov function method and mathematical analysis technique, sufficient conditions for achieving synchronization is first analytically derived. This result extends the condition derived for real-variable dynamical network to complex-variable network. Further, adaptive strategy is adopted to relax the restrictions on the impulsive intervals and reduce the control cost. Noticeably, the proposed adaptive pinning impulsive control scheme is universal for different dynamical networks to some extent. The impulsive instants are chosen by solving a series of maximum problems subject to the derived conditions. Several numerical simulations are performed to illustrate the effectiveness and correctness of the derived theoretical results.

1. Introduction

Many large-scale physical systems usually consist of large number of interacting individuals, which can be modeled by dynamical networks. The nodes of a network denote the individuals and the edges denote the interactions among them. In many practical applications, all the nodes of a network are desired to synchronize with a given orbit [1–24]. However, due to the complexities of node dynamics and topological structure of a network, all the nodes cannot achieve the goal themselves. That is, proper external controllers are required for achieving the goal. Recently, many control schemes are adopted to design effective controllers, such as adaptive control, intermittent control, impulsive control, pinning control, and so on.

Impulsive control is a kind of discontinuous control, i.e., the controllers are applied onto systems only at some discrete instants. Therefore, impulsive controllers are easy to implement and lower-cost since they have a relatively simple structure. Especially for those systems cannot endure continuous control, impulsive control scheme has been widely applied to design controllers. In most of the existing results about impulsive control, the largest impulsive interval is determined by some inequalities, which usually contain the Lipschitz or Lipschitz-like constant and the largest eigenvalue corresponding to the node dynamics and the topological structure of a network respectively. In other words, the estimated value of the largest impulsive interval may be conservative and much smaller than the needed value, which means that the control cost is larger than needed. Thus, how to relax the restrictions on the largest impulsive interval and reduce the control cost is an important and challenging problem. In [13], Chen et al. investigated synchronization of nonlinear chaotic systems by combining impulsive control and adaptive strategy. The Lipschitz constant need not to known beforehand for estimating the impulsive

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intervals, i.e., the control scheme relaxed the restrictions. Noticeably, for given impulsive intervals, the impulsive gains can adjust themselves to proper values for achieving synchronization.

As we know, controlling all the nodes of a network, especially those coupled with large number of nodes, is hard to implement and high-cost. Pinning control scheme, i.e., only a fraction of nodes are chosen to be controlled, has been extensively adopted to design proper controllers. In the past decade, many valuable results about pinning control have been obtained. In [19], Chen et al. investigated pinning synchronization of complex networks by a single controllers. In Refs. [14–16], authors investigated stabilization and synchronization of dynamical networks by combining pinning and impulsive control, and derived some conditions for estimating the impulsive intervals. Similarly, the estimated values of impulsive intervals may be conservative. Therefore, adaptive strategy should be adopted to design adaptive pinning impulsive controllers, which can make the impulsive intervals as large as possible.

On the other hand, many physical systems involving complex variable, which can be modeled by complex-variable dynamical systems. For example, the rotating fluids and detuned laser are described and simulated by complex Lorenz system [25,26]. Synchronization and control of coupled complex-variable chaotic systems have been widely studied and some valuable results have been obtained [27–33]. In [30], Wu et al. investigated the synchronization of a network coupled with complex-variable chaotic systems via adaptive feedback control and intermittent control respectively. To the best of our knowledge, the pinning impulsive synchronization of complex-variable dynamical network is seldom studied. On the other hand, pinning and impulsive control schemes are both low-cost and easier implement, and have been widely adopted in real-variable dynamical networks. That is, the synchronization of complex-variable dynamical networks via pinning impulsive control is deserved to be studied.

Motivated by the above discussions, this paper investigates the pinning impulsive synchronization of complex-variable dynamical network. Firstly, we extend the results obtained in Ref. [15] about real-variable network to complex-variable network. Secondly, we integrate adaptive strategy into the pinning impulsive control scheme for designing universal controllers to some extent. Based on the Lyapunov function method and mathematical analysis technique, we derive two synchronization criteria analytically. Furthermore, according to the adaptive strategy, we choose the impulsive instants by solving some maximum problems. Section 2 introduces the mathematical model and some preliminaries. Section 3 considers the pinning impulsive synchronization of complex-variable dynamical network and derives the main results. Section 4 performs several numerical simulations to demonstrate the effectiveness of the proposed control schemes. Section 5 gives conclusions and discussions about this paper.

Notations: The following notations will be used throughout this paper. For any complex number (or complex vector) $x, \bar{x}$ denotes the conjugate of x, $\|x\| = x^T\bar{x}$ dentes the norm of x.

2. Model and preliminaries

Consider a complex-variable dynamical network consisting of N complex-variable chaotic systems with linear diffusive coupling, which can be described by

$$\dot{x}_k(t) = f(x_k(t)) + \varepsilon\sum_{l=1}^{N} c_{kl}\Gamma x_l(t), \quad k = 1, 2, \ldots, N, \tag{1}$$

where $x_k(t) = (x_{k1}(t), x_{k2}(t), \ldots, x_{kn}(t))^T \in \mathbb{C}^n,\ f : \mathbb{C}^n \to \mathbb{C}^n$ is a nonlinear complex-valued vector function, $\varepsilon > 0$ is the coupling strength, $\Gamma = \mathrm{diag}(\gamma_1, \ldots, \gamma_n) \in \mathbb{R}^{n\times n}$ is the inner coupling matrix. $C = (c_{kl}) \in \mathbb{R}^{N\times N}$ is the zero-row-sum outer coupling matrix representing the topological structure of the network, defined as: if node k is affected by node $l (l \neq k)$, then $c_{kl} \neq 0$; otherwise, $c_{kl} = 0$.

The network (1) is said to achieve synchronization, if $\lim_{t\to\infty}\|x_k(t) - s(t)\| = 0$, where $s(t)$ is a solution of an isolated node, i.e., $\dot{s}(t) = f(s(t))$.

For achieving the synchronization, some pinning impulsive controllers are designed and applied onto a fraction of the nodes. The controlled network can be written as the following form:

$$\begin{aligned} &\dot{x}_k(t) = f(x_k, \theta) + \varepsilon\sum_{l=1}^{N} c_{kl}\Gamma x_l(t), \quad t \neq t_\sigma, \\ &x_k(t_\sigma^+) = x_k(t_\sigma^-) + B_k(t_\sigma)(x_k(t_\sigma^-) - s(t_\sigma^-)), \quad t = t_\sigma, \end{aligned} \tag{2}$$

where $k = 1, 2, \ldots, N,\ \sigma = 1, 2, 3, \ldots$, the impulsive time instants t_σ satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_\sigma < \cdots, t_\sigma \to \infty$ as $\sigma \to \infty,\ x_k(t_\sigma^+) = \lim_{t\to t_\sigma^+} x_k(t),\ x_k(t_\sigma^-) = \lim_{t\to t_\sigma^-} x_k(t)$. Any solutions of (2) are assumed to be left continuous at each t_σ, i.e., $x_k(t_\sigma^-) = x_k(t_\sigma)$. $B_k(t_\sigma)$ $(k = 1, 2, \ldots, N)$ are impulsive gains at $t = t_\sigma$ and $B_k(t) = 0$ for $t \neq t_\sigma$.

Let $e_k(t) = x_k(t) - s(t)$ be the synchronization errors, one can obtain the following error system:

$$\begin{aligned} &\dot{e}_k(t) = f(x_k(t)) - f(s(t)) + \varepsilon\sum_{l=1}^{N} c_{kl}\Gamma e_l(t), \quad t \neq t_\sigma, \\ &e_k(t_\sigma^+) = e_k(t_\sigma^-) + B_k(t_\sigma)e_k(t_\sigma^-), \quad t = t_\sigma, \end{aligned} \tag{3}$$