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A stochastic delay model for pricing debt and equity: Numerical techniques and applications

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ABSTRACT

Delayed nonlinear models for pricing corporate liabilities and European options were recently developed. Using self-financed strategy and duplication we were able to derive a Random Partial Differential Equation (RPDE) whose solutions describe the evolution of debt and equity values of a corporate in the last delay period interval in the accompanied paper (Kemajou et al., 2012) [14]. In this paper, we provide robust numerical techniques to solve the delayed nonlinear model for the corporate value, along with the corresponding RPDEs modeling the debt and equity values of the corporate.

Using financial data from some firms, we forecast and compare numerical solutions from both the nonlinear delayed model and classical Merton model with the real corporate data. From this comparison, it comes up that in corporate finance the past dependence of the firm value process may be an important feature and therefore should not be ignored.

1. Introduction

Due to the remarkable growth of the credit derivatives market, the interest in corporate claim value models and risk structure has recently increased. Financial distress tends to be an important factor in many corporate decisions. The two main sources of financial distress are corporate illiquidity and insolvency. In [1], Gryglewicz explains how changes in solvency affect liquidity and also how liquidity concerns affect solvency via capital structure choice. Corporate solvency is the ability to cover debt obligations in the long run. Uncertainty about average future profitability, with financial leverage, generates solvency concerns. Corporate insolvency may lead to corporate reorganization or to bankruptcy of the firm in the worst case. Corporate bankruptcy is central in the theory of the firm.[1] A firm is generally considered bankrupt when it cannot meet a current payment on a debt obligation. In this event, the equity holders lose all claims on the firm, and the remaining loss which is the difference between the face value of the fixed claims and the market value of the firm, is supported by the debt holders.

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[1] The theory of the firm is the set of economic theories that describe, explain, and predict the nature of the firm, including its existence, behavior, structure, and relationship to the market [26].

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In the literature of corporate finance, [7] appears to be the main pioneers in the derivation of formulas for corporate claims. This model is a dual of Black and Scholes model (see [10]) for stock price. Merton in [7] further analyzed the risk structure of interest rates. More specifically, he found the relation between corporate bond spreads and government bond, and attempted to determine a valid measure of risk. He also developed the deterministic partial differential equation modeling the debt and equity of the firm. The assumption of constant volatility in the original Black–Scholes and Merton models from which most claims derivations are inspired, is incompatible with derivatives prices observed in the market (see [3,4,6,5] and the references therein).

For stock price, two alternative theories are mostly used to overcome the constant volatility drawback. The first approach sometimes called *level-dependent volatility* describes the stock price as a diffusion with level-dependent volatility (see [9]). The second approach sometimes called *stochastic volatility* defines the volatility as an autonomous diffusion driven by a second Brownian motion.[2] In [15], a new class of nonconstant volatility model which can be extended to include the first of the above approaches, that we called *delayed model* is introduced and further studied in [8,14] for options prices. This model shows that the past dependence[3] of the stock price process is an important feature and therefore should not be ignored. The main goal of this model is to make volatility self-reinforcing. Since the volatility is defined in terms of past behavior of the asset price, the self-reinforcing is high[4] (see [15]).

This is designed to reflect real-world perceptions of market volatility, particularly if practitioners are to compare historic volatility with implied volatility.

Following the duality between the stock price [10] and corporate finance [7], we recently introduced in [11,12] the nonlinear delayed model for debt and loan guarantee. Using self-financed strategy and replication we established that debt value and equity value follow two related Random Partial Differential Equations (RPDEs) within the last delay period interval.[5] The analytical solution of the nonlinear model and RPDEs are unknown in general case and therefore numerical techniques are needed.

In recent years, the computational complexity of mathematical models employed in financial mathematics has witnessed a tremendous growth (see [20,19,13] and references therein). The aims of this paper is to solve numerically the delayed nonlinear model for firm market value along with the corresponding RPDEs, using real data from firms. Comparison will be done with classical Merton model. To the best of our knowledge such comparison has not yet been done in the financial literature. Furthermore our combined robust numerical method based on central finite difference-finite volume (discretization with respect to the firm value V) and exponential integrators (discretization with respect to the time t) has not yet used in finance. The finite volume method is used for space discretization of the convection term using standard upwinding technique while the central finite difference method is used for space discretization of the diffusion term. Comparing our combined method with the full discretization with finite volume method, the approximation of the diffusion term is more accurate (second order accuracy) while comparing with the full discretization with the central finite difference method, the approximation of the convection term is more stable. In our simulations, two major comparisons will be performed: the market value of each corporate and its equity value (or its debt value). We will first approximate the volatility of each corporate, afterward solve numerically the nonlinear model for the market value of the corporate along with the corresponding Merton model using the semi implicit Euler Maruyama scheme to obtain sample numerical solutions. The Monte Carlo method will be thereafter used to approximate the mean (the expectation) of the numerical samples solution of each model. The mean of numerical value from each model (the nonlinear model and Merton model) will be therefore compared with the real market value (V) of the corporate.

For equity value (or debt value), solutions of RPDEs established in [12], efficient numerical scheme based on central finite difference-finite volume method (discretization with respect to the firm value V) and exponential integrators (discretization with respect to the time t) will be used. Recently, exponential integrators have been used efficiently in many applications in porous media flow [2,18,17,23,24], but are not yet well spread in finance. The same numerical technique is also used to solve deterministic Partial Differential Equations (PDEs) modeling debt value or equity value in Merton model. Comparisons are done with the real data from firms for each model (the delayed model and Merton model).

From our comparison, it comes up that in corporate finance the past dependence[6] of the firm value process is an important feature and therefore should not be ignored. The main goal of this paper is to call for further attention into the possibility of modeling market value of the firm with nonlinear delayed stochastic differential equations.

The paper is organized as follows. In Section 2, we recall the delayed nonlinear model for corporate claims as presented in [12] along with the Merton model [7]. In Section 3, numerical techniques for the delayed nonlinear model are provided. We first present the semi implicit Euler Maruyama for the firm market value V and provide numerical experimentations for both the nonlinear model and Merton model using real data for some firms. We end this section by providing numerical technique to solve efficiently the (RPDEs) modeling the debt and equity of the firm along with numerical experimentations for the two models (the delayed nonlinear model and Merton model) with the real data from some firms. The conclusion is provided in Section 4.

[2] In the sense that the first Brownian motion drives the asset price.

[3] i.e. the distribution of future asset prices depends on past evolution of the asset prices.

[4] i.e. the volatility is more precise when it is a function (nonlinear or linear) of past stock price (or past expected return) and this precision is high comparing to the standard Black Scholes or Merton model (which use only the mean of the past expected return) when there been large movements in recent past.

[5] The final time interval with length equal to the time delay.

[6] i.e. the distribution of future firm value depends on past evolution as it was found for option prices.