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Review

Stochastic resonance in a single-well anharmonic oscillator with coexisting attractors

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ABSTRACT

We present a numerical investigation of occurrence of stochastic resonance in a single-well anharmonic oscillator where period doubling and chaotic orbits coexist with a large amplitude periodic orbit for a wide range of values of frequency ω of the external periodic force $f \sin \omega t$. Stochastic resonance occurs due to the noise-induced switching between the large amplitude periodic orbit and another coexisting orbit. The signal-to-noise ratio (SNR) is found to be maximum at an optimum value of noise intensity (D_{MAX}) and with ω, D_{MAX} increases while SNR at D_{MAX} decreases linearly in different rates with respect to the coexisting chaotic and periodic attractors. The mean residence times around the two coexisting orbits are not same at $D = D_{\mathrm{MAX}}$.

Contents

1. Introduction

The study of stochastic resonance (SR) has received an enormous interest in recent years. There were several theoretical developments of SR in double-well systems [1,2]. Very recently "SR splitting" in an optomechanical torsion oscillator confined to two asymmetric stable states due to the interplay of the inter-well and intra-well asymmetries built in the restoring potential of the torsion oscillator [3,4], double SR in an overdamped asymmetric double-well potential [5] and stochastic bifurcations and coherence-like resonance in a self-sustained bistable noisy oscillator [6] have been reported. Study of SR and other related noise-induced effects in monostable systems are very important because there are physical, chemical and biological systems modelled by single-well potential. In recent years there are investigations along this direction. We are concerned with SR in single-well systems.

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In monostable nonlinear systems with additive Gaussian white noise the signal-to-noise ratio (*SNR*) is shown to be always a decreasing function of input noise intensity [7]. In both linear and nonlinear systems with monostability, the addition of multiplicative noise is found to give rise bistability in the effective potential. As a result of this, SR is observed in nonlinear systems with symmetric [8] and asymmetric single-well potentials [9] driven by both additive and multiplicative noises. In certain monostable nonlinear systems SR is found to occur in high-frequency regime close to the natural frequency of the oscillator at the bottom of the well [10,11]. SR in three linear systems such as a horizontally driven pendulum, a forced electrical circuit and a laser with an injected signal have been analysed using multiplicative white noise [12]. SR is investigated in a single-well Duffing oscillator with additive noise which is driven at a frequency close to the natural frequency of the oscillator [13]. Existence of SR in some monostable systems are analysed using signal-to-noise ratio and its application has been discussed in ferromagnetic particles under an external magnetic field and also in a standard model for neuronal excitable medium [14]. Stochastic anti- and multi-resonance in an overdamped monostable system is studied [15]. An additive Lévy noise in a single-well nonlinear system is shown to induce a double SR [16]. On the other hand, in linear systems using theoretical approaches, SR is found due to multiplicative Gaussian noise [17,18], multiplicative asymmetric dichotomous noise [19], multiplicative coloured noise [20–22] both multiplicative and additive dichotomous noise [23,24], Poissonian noise [25] and signal modulated additive coloured noise [26].

In nonlinear systems coexistence of attractors is very common. Thus it is important to investigate the possibility of noise-induced switching between them in the context of SR. The focus of the present work is to study the occurrence of SR due to the switching between coexisting attractors in a single-well system. For this purpose we consider the anharmonic oscillator [27,28]

$$\ddot{x} + d\dot{x} + \omega_0^2 x + \alpha x^2 + \beta x^3 = f \sin \omega t + \sqrt{D}\eta(t), \tag{1}$$

where $\eta(t)$ is the Gaussian white noise with zero mean and autocorrelation $\langle \eta(t)\eta(t')\rangle = D\delta(t-t')$. We choose $\omega_0^2, \alpha, \beta > 0$ and $\alpha^2 < 4\beta\omega_0^2$ so that the potential of the system is in a single-well form. In the noise free system, for a wide region in (f, ω) parameter space, period doubling and chaotic orbits coexist with a large amplitude period-$T(= 2\pi/\omega)$ orbit. We consider the response of the system by varying the intensity D of the noise $\eta(t)$. For any choice of value of ω for which two attractors coexists, *SNR* first increases with D, reaches a maximum value at an optimum value of D and then decreases. This is a typical character of SR phenomenon. In the numerical simulation the optimum noise intensity D_{MAX}, at which *SNR* is maximum, increases linearly with ω in the chaotic as well as in the period doubling regions but with different rates. Since the amplitudes of the coexisting attractors are different, the mean residence time τ on each attractor is different. In the system (1) at $D = D_{MAX}$ the mean residence times around the two orbits are not equal to $T/2$ (but they are equal to $T/2$ in the symmetric double-well system) and further they are not same. The τ around the small orbit is always found to be greater than that of the large amplitude orbit. Also the probability distributions of normalised residence time around the two orbits at D_{MAX} are not same.

The paper is organised as follows. In Section 2 first we obtain the frequency–response equation from which the regions in parameter space where two different period-T orbits coexist can be found. We show the occurrence of SR in single-well case due to noise-induced switching between two coexisting attractors. We study the variations of D_{MAX} and SNR_{MAX} (the value of *SNR* at D_{MAX}) with ω and also the variation of τ on the two coexisting attractors with the noise intensity. We discuss the characteristics of distribution of normalised residence time. Section 3 contains concluding remarks.

2. Noise-induced switching between coexisting attractors

In the system (1) in the absence of noise more than one attractor coexist for a wide range of values of the parameters. For example, it has a large amplitude period-T orbit and a small amplitude period-T orbit for a certain range of values of the control parameter ω for fixed values of other parameters. When ω is varied a small amplitude orbit undergoes period doubling bifurcation leading to chaotic motion while the large amplitude orbit remains stable. The range of values of ω for which two period-T attractors coexist can be determined theoretically.

When the amplitude of the external force in (1) is weak we assume its solution in the limit $t \to \infty$ as

$$x(t) = B + A \sin(\omega t + \phi), \tag{2}$$

where A, B and ϕ are to be determined. Substituting the above solution in Eq. (1) without the noise term, equating the coefficients of $\sin \omega t, \cos \omega t$ and constants to zero separately and after simple mathematics we obtain

$$B^3 + \frac{\alpha}{\beta}B^2 + \left(\frac{\omega_0^2}{\beta} + \frac{3A^2}{2}\right)B + \frac{\alpha A^2}{2\beta} = 0 \tag{3}$$

and

$$\left[(\omega_0^2 - \omega^2)A + (2\alpha + 3\beta B)AB + \frac{3}{4}\beta A^3\right]^2 + (dA\omega)^2 = f^2. \tag{4}$$