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Detection of quasi-periodic processes in repeated measurements: New approach for the fitting and clusterization of different data

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ABSTRACT

Many experimentalists were accustomed to think that any independent measurement forms a non-correlated measurement that depends weakly from others. We are trying to reconsider this conventional point of view and prove that similar measurements form a strongly-correlated sequence of random functions with memory. In other words, successive measurements "remember" each other at least their nearest neighbors. This observation and justification on real data help to fit the wide set of data based on the Prony's function. The Prony's decomposition follows from the quasi-periodic (QP) properties of the measured functions and includes the Fourier transform as a partial case. New type of decomposition helps to obtain a specific amplitude–frequency response (AFR) of the measured (random) functions analyzed and each random function contains less number of the fitting parameters in comparison with its number of initial data points. Actually, the calculated AFR can be considered as the generalized Prony's spectrum (GPS), which will be extremely useful in cases where the simple model pretending on description of the measured data is *absent* but vital necessity of their quantitative description is remained. These possibilities open a new way for clusterization of the initial data and new information that is contained in these data gives a chance for their detailed analysis. The electron paramagnetic resonance (EPR) measurements realized for empty resonator (pure noise data) and resonator containing a sample (CeO_2 in our case) confirmed the existence of the QP processes in reality. But we think that the detection of the QP processes is a common feature of many repeated measurements and this new property of successive measurements can attract an attention of many experimentalists.

1. Introduction

In nowadays, it becomes evident that with the increasing of complexity of a system at different stages of its evolution, the fundamental and simple (from the mathematical point of view) rules that have been established earlier are difficult to find and then (if they were found) to justify. In order to understand better the behavior of the complex system irrespective to its specific features studied it is necessary to find some *general principles* that govern by a wide class of the complex system studied. Some principles are hidden inside and covered by the interruption of other uncontrollable factors known in

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measurements as an influence of random fluctuations or "noise". In order to decrease the influence of a random noise many experimentalists repeat their measurements many times having in mind the statistical paradigm: measurement repetition of the *uncorrelated* random sequence should decrease the amplitude of a noise at least in $1/N^{1/2}$ times. But hypothesis that noise is really uncorrelated is remained in many cases no more than as the unjustified *supposition*. Besides these hidden (in the most cases) relationships, there are other general rules that can characterize the degree of correlation between self-similar and other random sequences. These correlated relationships can serve as a specific indication to existence of a relevant law and on a certain stage help in understanding of a specific behavior of the complex system studied. The development of the fractal geometry [1,2] and its impact in the development of the mathematics of fractional calculus [3,4] allows establishing new mathematical relationships expressed in terms of the non-integer operators of differentiation and integration. In paper [5] the first author solved the inverse problem that existed for self-similar structures developing on different time scales. Nevertheless, we think that this self-similar (SS) scenario, in spite of its attractiveness and generality, is *not* unique. In this paper we want to show that there is another general principle which is submerged in a simple repetition of routine measurements. We will try to justify that a set of the repeated measurements has a memory and in the contrast to a conventional opinion they are strongly-correlated. In other words, these similar measurements remember each other and repetition of measurements in order to decrease their internal correlations is becoming *useless*. We define the principle governing by a temporal evolution of the repeated measurement process as the Quasi-Periodic (QP) process and describe its general properties. So, the problem that is considered in this work can be outlined as follows:

1. To formulate some general conditions that help to identify and then detect the presence of some QP process in the repeated experimental measurements.
2. To find a functional equation and its solution that yields the description of the identified QP process.
3. To suggest some computing algorithm for fitting of the QP data to the analytical function that follows from the solution of the corresponding functional equation.

The content of this paper is organized as follows. In the Section 2 we will try to find the answers on the problem posed in this introductory section. It contains also the mathematical description of the QP process and interpretation of the meaning of the generalized Prony's spectrum (GPS). The GPS includes the conventional Fourier decomposition as a *partial* case. Section 3 contains the experimental details associated with receiving of the desired data. Section 4 includes some important details explaining specific features of application of general algorithm to concrete data. In Section 5 we summarize the results and outline the perspectives of this approach for quantitative description of time-dependent random data that are registered in different complex systems and experimental devices. Here we should notice that under the *complex system* we imply a system when a conventional model is *absent* [6]. Under *simplicity* of the acceptable model we imply the proper hypothesis ("best fit" model) containing minimal number of the fitting parameters that describes the behavior of the system considered quantitatively. The different approaches that exist in nowadays for description of these systems are collected in the recent review [7].

2. Treatment procedure based on the generalized Prony decomposition

As it is well-known that a pure periodic process with the given period T satisfies to the following functional equation

$$\Pr(t \pm T) = \Pr(t). \tag{1}$$

The general solution of this functional equation is known and it can be expressed in the form of the Fourier series (if the initial function is defined on the discrete set of the given points $[t_j]\ j = 1, 2, \ldots, N$)

$$\Pr(t) = A_0 + \sum_{k=1}^{\infty}\left[Ac_k \cos\left(2\pi k \frac{t}{T}\right) + As_k \sin\left(2\pi k \frac{t}{T}\right)\right]. \tag{2}$$

Instead of Eq. (1) we consider somewhat generalized temporal process

$$F(t + T) = aF(t) + b, \tag{3}$$

where the parameters a and b determine some real constants. This functional equation means that: temporal evolution of some process taking place on the interval $t > T$ is based on events that took place presumably in the nearest past ($t < T$). This functional equation has been considered in the first time in paper [8] but in the present paper we want to expand it out of the scope of the previous consideration. The solution of this equation can be written in the following form [8]

$$\begin{aligned} &a \neq 1 : F(t) = \exp(\lambda t)\Pr(t) + c_0, \quad \lambda = \frac{\ln(a)}{T}, \quad c_0 = \frac{b}{1-a}, \\ &a = 1 : F(t) = \Pr(t) + b\frac{t}{T}. \end{aligned} \tag{4}$$

If $a > 1$ then we have the *increasing* exponential factor ($\lambda > 0$). The period T is supposed to accept the positive values. For this situation the influence of the past events on the present event is becoming *essential*. For $a < 1$ we have the effect of the