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Optimal control of information epidemics modeled as Maki Thompson rumors

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ABSTRACT

We model the spread of information in a homogeneously mixed population using the Maki Thompson rumor model. We formulate an optimal control problem, from the perspective of single campaigner, to maximize the spread of information when the campaign budget is fixed. Control signals, such as advertising in the mass media, attempt to convert ignorants and stiflers into spreaders. We show the existence of a solution to the optimal control problem when the campaigning incurs non-linear costs under the isoperimetric budget constraint. The solution employs Pontryagin's Minimum Principle and a modified version of forward backward sweep technique for numerical computation to accommodate the isoperimetric budget constraint. The techniques developed in this paper are general and can be applied to similar optimal control problems in other areas.

We have allowed the spreading rate of the information epidemic to vary over the campaign duration to model practical situations when the interest level of the population in the subject of the campaign changes with time. The shape of the optimal control signal is studied for different model parameters and spreading rate profiles. We have also studied the variation of the optimal campaigning costs with respect to various model parameters. Results indicate that, for some model parameters, significant improvements can be achieved by the optimal strategy compared to the static control strategy. The static strategy respects the same budget constraint as the optimal strategy and has a constant value throughout the campaign horizon. This work finds application in election and social awareness campaigns, product advertising, movie promotion and crowdfunding campaigns.

1. Introduction

Rumor models (*e.g.*, Daley Kendall, Maki Thompson) are used to model social contagion processes like spreading of information, ideas, fashion trends, etc. [1–3] in a population. A piece of information affects human behavior which may be exploited by political and crowdfunding campaigners, companies for advertising their new products etc. The goal of the campaigner is to reach as many people as possible by the campaign deadline, while making most efficient use of the available resources (*e.g.*, money, manpower). Since the contagion process is epidemic in nature, the allocation of resources over the campaign duration is important for optimal information spreading.

Rumor models are, in principle, similar to biological epidemic models like Susceptible-Infected-Susceptible (SIS) and Susceptible-Infected-Recovered (SIR), used for modeling the spread of pathogens in a population [4,5]. The population is

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divided into three compartments (or classes): ignorants (those who do not have the information), spreaders (those who are spreading the information) and stiflers (those who have stopped spreading). Spreaders are generated at some rate due to ignorant-spreader contact—dynamics which is similar to the biological epidemic models. However the recovery process in the rumor models is different from that in biological epidemic models, which is explained in the following.

Why rumor models are more appropriate than SIS/SIR models for information spreading: recovery of a spreader to a stifler is spontaneous and independent of others in SIS/SIR models. To be specific: if the recovery rate is γ, then a spreader recovers after a time duration which is exponentially distributed with mean $1/\gamma$, independent of her interaction with people in the population. On the other hand, in a rumor model, a spreader converts into a stifler at some rate, if she comes in contact with other spreaders or stiflers [1–3]. For a recovery rate γ, the quantity $1/\gamma$ provides a measure of the average number of interactions of a spreader with others, who are aware of the rumor, before she turns into a stifler [1, Section 10.2]. Information spreading is a psychological phenomenon, meeting others who already have the information changes the spreader's perception about information being new and she stops spreading. Due to this difference from SIS/SIR models, rumor models are more accurate in capturing information spreading dynamics. Similar arguments are applicable for fashion trends.

In this work, we aim to devise optimal information dissemination strategies, from the perspective of single campaigner, using the theory of optimal control. We assume that information spreading dynamics can be influenced by a control, which transfers individuals from the ignorant and stifler classes to the spreader class. Depending on the application—*e.g.*, political/crowdfunding campaigns; advertisement campaign for new products/services like smartphones, video games, satellite TV plans; fashion products like clothing, cosmetics—this can be done in various ways. Examples of ways in which control can be implemented in real systems include publishing manifestos, organizing political rallies/door-to-door campaigns, advertising in mass media and giving out discounts on new products, signing up brand ambassadors etc. When an ignorant comes across the advertisement, she becomes aware of the information and starts spreading. Also, when a stifler sees the advertisement, her perception about an information being stale or fashion/product being old changes, and she starts spreading the information again or following the fashion trend again. Note that the control acts in addition to the epidemic ignorant-spreader contact which transfers ignorants to the spreader class.

Readers should not be misled by the term 'rumor' in the Maki Thompson model. They can be used to model both useful and malicious information. In this work, we have used them for modeling only useful information and then attempt to maximize its reach. This paper does not address the problem of suppressing malicious information. Also note that apart from the direct applications listed in the previous paragraph, the tools and techniques developed in this paper can be used in other optimal control problems, such as mitigating the spread of biological epidemics and computer viruses, treating cancer [6] and suppressing corruption, terrorism and drug use [7].

Related work and differences compared to the previous literature: optimal control of SIS and SIR information epidemics was studied in [8,9]. This work employs a more accurate information diffusion model, namely Maki Thompson model, as explained above. In addition, this work has an explicit budget constraint and non-linear resource application costs, which differentiates it from [8,9]. The authors in [10,11] studied impulsive control strategies to maximize Maki Thompson rumors. The rumor starts with a broadcast, and then there is an opportunity to trigger a second broadcast at a later stage. The work in [10,11] determined the optimum time to trigger the second broadcast so that the number of ignorants in the system is minimized. In contrast, our formulation allows the system to be controlled throughout the campaign duration. In the applications considered—*e.g.*, political campaigns, product marketing, movie promotion—the campaigner tries to influence the system on a continuous basis. Advertisements appear in the mass media on a frequent basis, and not just once or twice; this motivates our model.

The work in [12] devises an optimal advertising and pricing plan for a newly launched product; however, it does not consider epidemic information diffusion in the population, as is the case in this paper. The authors in [13,14] analyze 'push', 'pull' and 'push/pull' strategies for message diffusion, where nodes in the network either push the information to their neighbors, or pull it from them. Their aim is not to control the system but to find bounds on the number of communication rounds required to spread the information to almost all nodes in the network.

Optimal control of disease and computer virus epidemics is a well studied problem [15–25]. In addition to the differences in the epidemic model, biological epidemics need to be contained, which is the opposite of spreading information. A biological epidemic has constant spreading rate, assuming the pathogens will not mutate within a season. In contrast, we have allowed the spreading rate to vary during the campaign duration to capture varying interest level of the population in the subject of the campaign during the time horizon of interest.

There is a sizeable literature on (uncontrolled) Maki Thompson rumor model and its extensions/generalizations. See for example, [26–30].

The following are the primary contributions of this paper:

(i) We have formulated and numerically solved the optimal control problem for maximizing information spread in the Maki Thompson model. In our formulation, the system can be controlled throughout the campaign duration, which is different from the impulsive control in [10,11]. The control directly recruits ignorants and stiflers to spread the information. This can be done via methods such as placing advertisements in mass media. We assume a non-linear cost for applying the control and a fixed budget constraint. The standard forward backward sweep method used to solve the problem numerically needs to be modified due to the isoperimetric budget constraint.