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Switching control and time-delay identification

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ABSTRACT

The unknown time delay makes the control design a difficult task. When the lower and upper bounds of an unknown time delay of dynamical systems are specified, one can design a supervisory control that switches among a set of controls designed for the sampled time delays in the given range so that the closed-loop system is stable and the control performance is maintained at a desirable level. In this paper, we propose to design a supervisory control to stabilize the system first. After the supervisory control converges, we start an algorithm to identify the unknown time delay, either on-line or off-line, with the known control being implemented. Examples are shown to demonstrate the stabilization and identification for linear time invariant and periodic systems with a single control time delay.

1. Introduction

Whenever there is uncertainty, switching controls are used to maintain proper performance of dynamical systems. Uncertainty can be associated with the lack of the knowledge of the system leading to the model with uncertain parameters or functional structures. When there is time delay in the system, the delay is often uncertain as well. This paper presents a study of switching control for dynamical systems with uncertain control delay and an identification algorithm for estimating the time delay.

There are many studies of control systems with unknown and time-varying time delays. Sufficient conditions for the existence of the guaranteed cost output-feedback controller in terms of matrix inequalities for uncertain dynamic systems with time delay are derived in [1]. A class of iterative learning controls with uncertain state delay and control delay have been studied in [2]. The stability problem is treated in the integral quadratic constraint (IQC) framework. Delay-dependent robust stability for neutral systems with the help of the Lyapunov method is investigated in [3]. Robust stability of systems with random time-varying delay is studied in [4]. Sufficient conditions for exponential mean square stability of the system are derived by using the Lyapunov method and the linear matrix inequality (LMI) technique.

The supervisory control proposes to use several estimates of uncertain parameters for the system model [5–8]. For each estimate of the parameter, a control is designed. A supervisor monitors the real-time response of the system, selects a plant model according to a switching criterion and implements the corresponding control. The supervisory control can be applied to systems with uncertain time delay when the uncertain time delay is bounded with known lower and upper bounds [9]. It

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turns out that the switching index of the supervisory control designed for the system with unknown time delay tracks very well the true time delay in linear time invariant systems.

The identifiability of time delay in linear systems has been studied in [10]. Genetic algorithms have been developed to estimate system parameters and time delays in the feedback at the same time for both linear and nonlinear systems in [11]. A particle swarm algorithm for optimization is applied to identify time delay of a structural system in [12]. The method of harmonic balance is extended to identify time delay of nonlinear systems in [13]. Herein, we present a two stage approach to identify the unknown time delay of a dynamical system. In the first stage, we apply the supervisory control to stabilize the system. Once the system is stabilized, an identification routine is then started to estimate the time delay either on-line or off-line.

The rest of the paper is outlined as follows. First, the supervisory control and switching rule are presented in Section 2 for the system with unknown time delays. The methods for control design of linear dynamical systems with known time delay are then reviewed in Section 3. An identification method for estimation of unknown time delay follows in Section 4. Section 5 presents numerical examples of switching control and time delay identification of linear dynamical systems with unknown time delay. Section 6 concludes the paper.

2. Switching control

Consider a dynamical system with time delay given by,

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t) + \mathbf{B}\mathbf{u}(t - \tau_c), \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in \mathbf{R}^m$, $\mathbf{f}$ describes the system dynamics with time delay, and $\mathbf{B} = \{\mathbf{B}_{ij}\}$ is the control influence matrix. τ_c is a control delay. Assume that the upper and lower bounds of the delay $[\tau_{c\min}, \tau_{c\max}]$ are known. We discretize $[\tau_{c\min}, \tau_{c\max}]$ into $M_c - 1$ intervals such that $\tau_{c\min} = \tau_{c1} < \tau_{c2} \cdots < \tau_{cM_c} = \tau_{c\max}$. Consider M_c models of the time-delayed system as

$$\dot{\mathbf{x}}_j = \mathbf{f}\left(\mathbf{x}_j(t), t\right) + \mathbf{B}\mathbf{u}_j\left(t - \tau_{cj}\right), \quad 1 \leqslant j \leqslant M_c. \tag{2}$$

Assume that we have obtained a set of controls $\mathbf{u}_j\left(t - \tau_{cj}\right)$ for the set of sampled time delays. The control $\mathbf{u}_j\left(t - \tau_{cj}\right)$ must be stable for all the time delay in the known range.

Following the concept of the supervisory control [5–8], we define an estimation error as

$$\mathbf{e}_j = \mathbf{x}_j(t) - \mathbf{x}(t), \quad 1 \leqslant j \leqslant M_c, \tag{3}$$

where $\mathbf{x}(t)$ is the output of the system with unknown time delay. In experiments, $\mathbf{x}(t)$ would be obtained from measurements. Consider a positive function of the estimation error $F_j(\mathbf{e}_j) > 0$. A good example is $F_j(\mathbf{e}_j) = ||\mathbf{e}_j||^2$. Define a switching index $\pi_j(t)$ such that

$$\begin{aligned} &\dot{\pi}_j(t) + \lambda_j \pi_j(t) = F_j(\mathbf{e}_j), \quad (\lambda_j > 0) \\ &\pi_j(0) = 0, \end{aligned} \tag{4}$$

where the parameter λ_j defines the bandwidth of the low pass filter.

The hysteretic switching algorithm in [7,8] is stated as follows. Assume that the system is sampled at a fixed time interval. At the k^{th} time step, the system is under control $\mathbf{u}_j\left(t - \tau_{cj}\right)$ and the associated switching signal is $\pi_j(k)$. At the $(k+1)th$ step, if there is an index l such that $\pi_l(k) < (1 - \eta)\pi_j(k)$ where $\eta > 0$ is a small number, we switch to the control $\mathbf{u}_l(t - \tau_{cl})$. Otherwise, we continue with the same control. η is known as the hysteretic parameter and prevents the system from switching too frequently.

In [9], the supervisory control has been studied for linear time-invariant as well as periodic systems with uncertain time delay where the lower order controls have been considered. It has been found that the switching algorithm can lead the controlled system to one with the best performance in terms of the switching index. In the meantime, the associated time delay of the converged control tracks the unknown time delay for linear time invariant systems, and sometimes for periodic systems. Finally, it is noted that many methods can be used to design controls in order to apply the hysteretic switching rule. In the following, two methods used in this work are reviewed.

3. Control design

In the following discussion, all the quantities would carry the subscript j for the sampled time delays τ_{cj}. For brevity, we shall omit the subscript j.

3.1. Optimal feedback gains via mapping

In [14,15], a mapping method to design optimal gains has been studied extensively. The mapping of an extended state vector involving the delayed response can be constructed by the method of semi-discretization or continuous time approximation. For linear time-invariant as well as periodic systems of order n [16–19], the mapping reads