Commun Nonlinear Sci Numer Simulat 19 (2014) 4196–4212

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Multi-harmonic measurements and numerical simulations of nonlinear vibrations of a beam with non-ideal boundary conditions

M. Claeys [a,b], J.-J. Sinou [b,*], J.-P. Lambelin [a], B. Alcoverro [a]

[a] *CEA/CESTA, CS60001, 15 avenue des Salinières, 33116 Le Barp, France*

[b] *Laboratoire de Tribologie et Dynamique des Systèmes, UMR-CNRS 5513, Ecole Centrale de Lyon, 36 avenue Guy de Collongue, 69134 Ecully Cedex, France*

ARTICLE INFO

Article history:
Received 5 November 2013
Received in revised form 4 April 2014
Accepted 8 April 2014
Available online 24 April 2014

Keywords:
Nonlinear vibrations
Experiments and numerical simulations
Nonlinear methods
Beam with non-ideal boundary conditions

ABSTRACT

This study presents a direct comparison of measured and predicted nonlinear vibrations of a clamped–clamped steel beam with non-ideal boundary conditions. A multi-harmonic comparison of simulations with measurements is performed in the vicinity of the primary resonance. First of all, a nonlinear analytical model of the beam is developed taking into account non-ideal boundary conditions. Three simulation methods are implemented to investigate the nonlinear behavior of the clamped–clamped beam. The method of multiple scales is used to compute an analytical expression of the frequency response which enables an easy updating of the model. Then, two numerical methods, the Harmonic Balance Method and a time-integration method with shooting algorithm, are employed and compared one with each other. The Harmonic Balance Method enables to simulate the vibrational stationary response of a nonlinear system projected on several harmonics. This study then proposes a method to compare numerical simulations with measurements of all these harmonics. A signal analysis tool is developed to extract the system harmonics' frequency responses from the temporal signal of a swept sine experiment. An evolutionary updating algorithm (Covariance Matrix Adaptation Evolution Strategy), coupled with highly selective filters is used to identify both fundamental frequency and harmonic amplitudes in the temporal signal, at every moment. This tool enables to extract the harmonic amplitudes of the output signal as well as the input signal. The input of the Harmonic Balance Method can then be either an ideal mono-harmonic signal or a multi-harmonic experimental signal. Finally, the present work focuses on the comparison of experimental and simulated results. From experimental output harmonics and numerical simulations, it is shown that it is possible to distinguish the nonlinearities of the clamped–clamped beam and the effect of the non-ideal input signal.

1. Introduction

The frequency response of mechanical structures often exhibit nonlinear behaviors such as dependency of eigenfrequencies and dissipation with input amplitude, discontinuities and hysteresis in the frequency response, multi-harmonic response to a mono-harmonic excitation. The sources of these nonlinearities are well known and various simulation

* Corresponding author. Tel.: +33 4 72 18 64 66.
E-mail address: jean-jacques.sinou@ec-lyon.fr (J.-J. Sinou).

http://dx.doi.org/10.1016/j.cnsns.2014.04.008
1007-5704/

methods have been developed to compute a nonlinear frequency response [1,2]. Yet the first applications of nonlinear simulation to industrial structures are very recent. In particular Renson and Kerschen [3] applied a time-integration method with a shooting algorithm [4] to compute the nonlinear vibrations of the SmallSat spacecraft, and focused on internal resonances between linear modes. Sinou [5] applied a Harmonic Balance Method to a nonlinear model of an industrial rotor and computed a multi-harmonic frequency response.

Beams are among the few systems whose nonlinear vibrations have been studied both experimentally and theoretically. Nayfeh and Mook [1] studied nonlinear vibrations of beams under several boundary conditions and showed how these dynamical problems can be treated by the method of multiple scales. Tabaddor [6] pointed out the importance of non-ideal boundary conditions in the modeling of beams. Malatkar and Nayfeh [7] explained how the method of multiple scales enables an easy identification of the nonlinear parameters. This identification method cannot be used on an industrial complex system, but fits well the needs of this study. Kerschen et al. [8] have written an extensive review of nonlinear system identification methods. The two nonlinear methods previously introduced with their industrial applications have also been used for the study of clamped–clamped beam nonlinear vibrations [9–11].

The main objective of this study is to propose a global strategy based on experiments and simulations for identifying the non-ideal boundary conditions and for predicting the nonlinear vibrations of the system that is subjected to multi-harmonic excitations. This global strategy consists in three steps. First of all, the method of multiple scales is used to update the non-ideal boundary conditions of the nonlinear beam with a mono-harmonic excitation. Secondly, the Harmonic Balance Method is developed to compute easily the multi-harmonic frequency response of the system to multi-harmonic excitations. Finally, the third step consists in comparing the multi-harmonic response with experiments. Such a comparison needs an efficient signal processing tool to extract the experimental multi-harmonic frequency response from a temporal signal. This paper proposes and presents a process based on high-selective filtering and fitting to synthetic functions using the Covariance Matrix Adaptation Evolutionary Strategy (CMAES) [12].

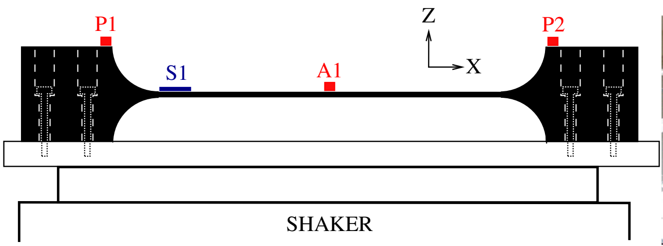
In addition, the Harmonic Balance Method is compared to the time-integration method with a shooting algorithm [4], to demonstrate the efficiency of both methods, and complementary experimental measures are performed to substantiate the model developed.

The next section of this article presents the clamped–clamped beam experiment with the method used to compute the experimental multi-harmonic frequency response. Then the beam model with non-ideal boundary conditions is detailed. The fourth section of this article focuses on the updating of the non-ideal boundary conditions by the method of multiple scales. The two numerical methods are explained in the fifth section. Finally, the multi-harmonic comparison between experimental measures and simulations is presented and interpreted. The simulations are made both with an ideal mono-harmonic input and with the experimental multi-harmonic input.

2. Experiments

2.1. Experimental setup

The experimental setup is presented in Fig. 1. The system studied is a steel beam of dimension $470 \times 20 \times 5$ mm. At both ends, the beam is bonded to a heavy steel block, of dimension $100 \times 100 \times 85$ mm. The whole piece (beam and blocks) is manufactured from a unique bulk piece of steel. Hollows between the beam and the blocks are designed to avoid stress concentration. The blocks are screwed on a large circular aluminum plate, itself screwed on the shaker. The system is instrumented with a strain gage and 3 three-dimensional accelerometers as depicted in Fig. 1. Each signal is denoted by the sensor name followed by the direction of the measurement. The entrance signal $\frac{1}{2}(P1Z + P2Z)$ is the shaker's feedback control signal. The reference frame of the study is the one of the blocks, where P1 and P2 are fixed. The zero-point of this reference frame coincides with A1 when the system is at rest. The response signal is the acceleration at the middle of the beam, in the reference frame of the blocks, that is $A1Z - \frac{1}{2}(P1Z + P2Z)$. The horizontal displacement of the blocks is also measured with the signal $\frac{1}{2}(P2X - P1X)$. The strain gage is used to evaluate the static constraints in the beam (see Section 2.2). Increasing and decreasing swept sine experiments in the vicinity of the primary resonance is realized.



(a)

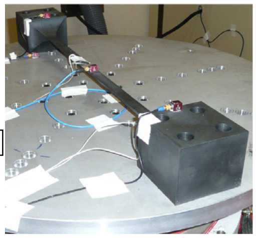

(b)

Fig. 1. A schematic (a) and a picture (b) of the experimental setup which includes 3 accelerometers (P1, P2, A1) and a strain gage (S1).