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Center and isochronous center conditions for switching systems associated with elementary singular points

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ABSTRACT

In this paper, an existing method is modified for computing the focal values and period constants of switching systems associated with elementary singular points. In particular, a quadratic switching system is considered to illustrate the computational efficiency of this method. Further, with this method, a cubic switching system is constructed to show existence of 15 limit cycles, which is the best result so far obtained for cubic switching systems.

1. Introduction

As one of the most important bifurcation phenomena, Hopf bifurcation plays an important role in the study of nonlinear dynamical systems. Many results on Hopf bifurcation for continuous systems have been obtained, especially for planar differential systems, see for example [1–3]. As far as the maximal number of small-amplitude limit cycles, bifurcating from an elementary center or focus, is concerned, the best known result is $M(2) = 3$, obtained by Bautin in 1952 [4]. Here, $M(n)$ denotes the maximal number of small-amplitude limit cycles around a singular point with n being the degree of polynomials in the vector field. For $n = 3$, a number of results have been obtained. Around an elemental focus, James and Lloyd [5] considered a special class of cubic systems to obtain 8 limit cycles in 1991, and the systems were reinvestigated couple of years later by Ning et al. [6] to find another solution of 8 limit cycles. Yu and Corless [7] constructed a cubic system and combined symbolic and numerical computations to show 9 limit cycles in 2009, which was confirmed by purely symbolic computation with all real solutions obtained in 2013 [8]. Another cubic system was also recently constructed by Lloyd and Pearson [9] to show 9 limit cycles with purely symbolic computation. Recently, Yu and Tian [10] have shown that there can exist 12 limit cycles around an elementary center in a planar cubic-degree polynomial system. This is the best result obtained so far for cubic polynomial systems with all limit cycles around a single singular point. For $n \geq 4$, there are very few results, for example, Huang gave an example of a quartic system with 8 limit cycles bifurcating from a fine focus [11].

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However, in modeling practical physical and engineering problems, there exist many problems which involve discontinuous or non-smooth functions, see for instance [12] and [13], and the references therein. Such examples include relay feedback systems in control theory [14,15], switching circuits in power electronics [16], impact and dry frictions in mechanical engineering [17,18], etc. In recent years, study of switching systems associated with Hopf bifurcation has attracted many researchers. Leine and Nijmeijer [19], and Zou et al. [20] considered non-smooth Hopf bifurcation. Freire et al. [21] discussed the focus-center limit cycle bifurcation in a symmetric three-dimensional, piecewise linear system. For homoclinic bifurcation, the Melnikov function method has been extended to study non-smooth systems [22,23]. General effective methods have also been developed to investigate non-smooth systems. For example, normal form computation for impact oscillators was given in [24], and a general methodology for reducing multidimensional flows to low dimensional maps in piecewise nonlinear oscillators was proposed in [25]. Due to complexity in non-smooth systems, such systems can exhibit not only all types of bifurcations that occur in smooth systems, but also complicated nonstandard bifurcation phenomena that are unique in non-smooth ones, such as grazing [26,27], sliding effects [17], border collision [28], etc. There are many articles in the literature, devoted to study various nonstandard bifurcations for non-smooth systems; see, for example, [17,18,26–29] and the references therein.

Recently, Chen and Du constructed a quadratic switching system to obtain 9 limit cycles [30]. Llibre et al. studied the maximum number of limit cycles that bifurcate from the periodic orbits of isochronous centers in switching cubic polynomial differential systems [31] and in switching quadratic polynomial differential systems [32]. These examples show that there exist more limit cycles in switching systems than in continuous systems, and the dynamics of these systems are more complex.

In this paper, the switching planar system, described by the following ordinary differential equations:

$$\begin{cases} \dfrac{dx}{dt} = -y + F^+(x,y), \\ \dfrac{dy}{dt} = x + G^+(x,y), \end{cases} (y > 0), \quad \begin{cases} \dfrac{dx}{dt} = -y + F^-(x,y), \\ \dfrac{dy}{dt} = x + G^-(x,y), \end{cases} (y < 0), \tag{1.1}$$

will be used to investigate bifurcation of limit cycles. In particular, a method for computing the Lyapunov constants of system (1.1) is present in Section 2, and then an approach for computing the period constants of system (1.1) is given in Section 3. Then, in Section 4 a quadratic switching system, as an example, is given to illustrate the computation efficiency of our methods; and further in the same section we construct a cubic switching system to show that system (1.1) can exhibit at least 15 limit cycles, which is a new best result for such systems. Finally, conclusion is drawn in Section 5.

2. Computation of Lyapunov constants of system (1.1)

In this section, we present a method for computing the Lyapunov constants of the switching system (1.1). First, we introduce some basic formulas of computing Lyapunov constants and period constants. The classical method to solve center problems is based on computing Lyapunov constants, with the procedure described as follows.

The general differential system,

$$\begin{aligned} \frac{dx}{dt} &= \delta x - y + \sum_{k=2}^{n} X_k(x,y) \equiv X(x,y), \\ \frac{dy}{dt} &= x + \delta y + \sum_{k=2}^{n} Y_k(x,y) \equiv Y(x,y), \end{aligned} \tag{2.1}$$

under the polar coordinates transformation,

$$x = r\cos\theta, \quad y = r\sin\theta, \tag{2.2}$$

can be rewritten as

$$\begin{aligned} \frac{dr}{dt} &= r\left(\delta + \sum_{k=2}^{n} \varphi_{k+2}(\theta) r^k\right), \\ \frac{d\theta}{dt} &= 1 + \sum_{k=2}^{n} \psi_{k+2}(\theta) r^k, \end{aligned} \tag{2.3}$$

where $\varphi_k(\theta)$, $\psi_k(\theta)$ are polynomials of $\cos\theta$ and $\sin\theta$, given by

$$\begin{aligned} \varphi_k(\theta) &= \cos\theta X_{k-1}(\cos\theta, \sin\theta) + \sin\theta Y_{k-1}(\cos\theta, \sin\theta), \\ \psi_k(\theta) &= \cos\theta Y_{k-1}(\cos\theta, \sin\theta) - \sin\theta X_{k-1}(\cos\theta, \sin\theta). \end{aligned}$$

From Eq. (2.3) we have

$$\frac{dr}{d\theta} = \frac{r(\delta + \sum_{k=2}^{n} \varphi_{k+2}(\theta) r^k)}{1 + \sum_{k=2}^{n} \psi_{k+2}(\theta) r^k}, \tag{2.4}$$

whose expansion around $r = 0$ can be expressed in the form of

$$\frac{dr}{d\theta} = r \sum_{k=1}^{\infty} R_k(\theta) r^k. \tag{2.5}$$