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Dynamical behavior of a one-prey two-predator model with random perturbations

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ABSTRACT

The objective of this paper is to systematically study the qualitative properties of a stochastic one-prey two-predator model. We have derived sufficient conditions (parametric restrictions) for extinction of each species and at the same time we also notice that when one or two species go extinction, remaining species can be stable in time average under same parametric restrictions, *i.e.*, extinction of one or two species ensures about the stability in mean of other species. Next, we have proved that the system admits a stationary distribution under some simple parametric conditions, which can be considered as a stability of the system in weak sense. Finally, we have proved that system is globally asymptotically stable.

1. Introduction

Predator–prey interaction is one of the important phenomena in ecology. During the interaction of two ecological species if the growth rate of one species increases while the growth rate of other species decreases, then we consider the interaction as predator–prey type interaction. In [1] Tripathi et al. gave an interesting example of predator–prey system, which is the snowy owl, that feeds almost exclusively on the common Arctic rodent called lemming while the lemming uses Arctic tundra plant for its food supply. The first mathematical model describing the predator–prey interaction is Lotka–Volterra model, which was proposed to explain the oscillatory levels of certain fish in the Adriatic sea during the first world war (for details one can see [2]). Several people used this model as a base model to formulate another model of two species competitive system [2,3]. From last few decade, many researchers investigated Lotka–Volterra model and they studied different kinds of dynamical properties like stability of positive periodic solution, stability of equilibrium points etc. [2–6] in detail. On the other hand, Volterra first concluded that existence of a system consisting of an odd number of species is not possible and it could be reduced to a system containing an even number of species. But, this conclusion contradicts many real world situations. Hence, three dimensional competition models have been extensively discussed by several researchers. Parish and Saila [7], Cramer and May [8], Koch [9], May and Leonard [10], Freedman and Waltman [11], Pande [12], Lin and Kahn [13], Hallam et al. [14], Bhat and Pande [15], Shukla and Das [16], Hutson and Vickers [17], Hutson and Law [18] and Roy and Solimano [19] investigated deterministic models for three interacting species and discussed conditions for co-existence, stability of the system and their equilibrium solutions.

Hsu et al. [20] analyzed a one-prey two-predator model. Their analytical results identified large regions of parameter space where the principle of competitive exclusion holds, that is, where one of the predators tends to extinction with increasing time. In [21], Freedman and Waltman considered a general three species predator–prey system called Kolmogorov system and discussed

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the persistence behavior of the system. Farkas and Freedman [22] derived sufficient conditions for stability of a one-prey two-predator system and Dubey and Upadhyay [23] studied persistence and extinction of another one-prey two-predators system.

But, the deterministic model neglects the random variation of environment, which is integral part of any realistic population model. Hence, stochastic models are more appropriate for describing the population process. Generally, stochastic model can be analyzed completely when the population size is very small. Stochastic models for complex ecological systems describing the growth and interaction of two ecological species were first introduced by Chiang in [24]. In [25], Bartlett observed the ultimate extinction scenario in two dimensional stochastic prey–predator models. Leslie [26] formulated stochastic models for both single species and two interacting species and discussed their properties using numerical simulations. Leslie and Gower [27] considered a stochastic model of prey–predator system and examined its properties within a limited environment where assumption of sufficient food supply for prey is considered. Also, Tsokos and Hinkley [28] studied the properties of a general bivariate stochastic model by deriving the estimation for its parameters. In [29], Gard and Kannan formulated a stochastic differential equation (SDE) model based on deterministic prey–predator model and studied the extinction property for each population.

In the present work, we consider the behavior of a predator–prey system consisting of two predator species, x_2 and x_3, and a single prey species, x_1. Here, we assume that each species compete with each other at different rates with no interference between rivals (no toxins are produced, for example). Both predator species can access the prey with different rates. The growth rates are proportional to the number currently alive. We also assume that there are no significant time lags in the system and the growth rates are logistic in each species in the absence of other two species. The model is given by

$$\begin{cases} \dfrac{dx_1(t)}{dt} = x_1(t)[r_1 - a_{11}x_1(t) - a_{12}x_2(t) - a_{13}x_3(t)], \\ \dfrac{dx_2(t)}{dt} = x_2(t)[r_2 - a_{21}x_1(t) - a_{22}x_2(t) - a_{23}x_3(t)], \\ \dfrac{dx_3(t)}{dt} = x_3(t)[r_3 - a_{31}x_1(t) - a_{32}x_2(t) - a_{33}x_3(t)], \end{cases} \tag{1}$$

where $x_2(t)$ and $x_3(t)$ are the number of predators at time t, $x_1(t)$ is the number of the prey at time t, r_1 is the growth rate (birth rate) of prey and r_2, r_3 are growth rates (death rates) for predator species, a_{ii} for $i = 1, 2, 3$ are intraspecific competition rates and a_{ij} for $i \neq j$, $i, j = 1, 2, 3$ are interspecific competition rates for prey and predator species, respectively. r_2, r_3, a_{21} and a_{31} are negative while other coefficients are positive.

The above deterministic model has been well studied by Gopalsamy [30] and Freedman and Waltman [31]. Freedman and Waltman obtained criteria for system to be persistent while Gopalsamy established some sufficient conditions for the stability of positive equilibrium.

But, the effect of environmental fluctuation on model system (1) was not observed by anyone so far. To capture this effect it is necessary to consider the stochastic differential equation (SDE) model corresponding to the deterministic model (1). Since the system parameters also vary randomly due to the environmental fluctuations, we will formulate the SDE model based on the model (1) by considering the system parameters as random in nature.

Suppose that the environmental noises mainly affect the growth rate parameters, with (see, e.g., [32–45]):

$$r_i \to r_i + \sigma_i dB_i(t),$$

where $B_1(t)$, $B_2(t)$, and $B_3(t)$ are standard mutually independent Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, σ_i is the intensity of the white noise, $1 \leq i \leq 3$. Then we obtain the following stochastic differential equation (SDE) system:

$$\begin{cases} dx_1(t) = x_1(t)[r_1 - a_{11}x_1(t) - a_{12}x_2(t) - a_{13}x_3(t)]dt + \sigma_1 x_1(t)\, dB_1(t), \\ dx_2(t) = x_2(t)[r_2 - a_{21}x_1(t) - a_{22}x_2(t) - a_{23}x_3(t)]dt + \sigma_2 x_2(t)\, dB_2(t), \\ dx_3(t) = x_3(t)[r_3 - a_{31}x_1(t) - a_{32}x_2(t) - a_{33}x_3(t)]dt + \sigma_3 x_3(t)\, dB_3(t). \end{cases} \tag{2}$$

For model (2), some interesting and important topics are as follows.

(i) System (2) is a stochastic population model, it is important and interesting to study the persistence and extinction of each species, and then to reveal the effects of stochastic noises on the persistence and extinction of each species.
(ii) In the study of population models, stability of positive equilibrium point is one of the most interesting topics. However, system (2) has no positive equilibrium point. Therefore, the solution of system (2) cannot tend to a fixed positive point. Hence, it is interesting to investigate the future state of the system, whether it will converge to some stationary state or exhibit non-equilibrium fluctuation without settling down.
(iii) Moreover, global asymptotic stability (global attractivity) of positive solution is an interesting problem in the study of population dynamics. Then when the solution of Eq. (2) is globally asymptotically stable?

The main objective of this paper is to consider the above questions. In Section 2, we study persistence and extinction properties of system (2). We prove that for every $0 \leq i \leq 3$, if the coefficients obey certain conditions, then $3 - i$ populations go to extinction almost surely, whilst the remaining i components tend to a positive constant in the sense of time average almost surely, i.e.,

$$\lim_{t \to +\infty} t^{-1} \int_0^t x_i(s)ds = \text{a positive constant}.$$