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# Learning monopolies with delayed feedback on price expectations ☆

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We call the intercept of the price function with the vertical axis the *maximum price* and the slope of the price function the *marginal price*. In this paper it is assumed that a monopolistic firm has full information about the marginal price and its own cost function but is uncertain on the maximum price. However, by repeated interaction with the market, the obtained price observations give a basis for an adaptive learning process of the maximum price. It is also assumed that the price observations have fixed delays, so the learning process can be described by a delayed differential equation. In the cases of one or two delays, the asymptotic behavior of the resulting dynamic process is examined, stability conditions are derived. Three main results are demonstrated in the two delay learning processes. First, it is possible to stabilize the equilibrium which is unstable in the one delay model. Second, complex dynamics involving chaos, which is impossible in the one delay model, can emerge. Third, alternations of stability and instability (i.e., stability switches) occur repeatedly.



## 1. Introduction

This paper considers the familiar monopoly model in a market with linear demand and the only firm has linear cost. Implicit in the text-book approach is an assumption of complete and instantaneous information availability on price and cost functions. In consequence, the textbook-monopoly can find its optimal choices of price and quantity to maximize profit with one shot. Thus the traditional monopoly model is *static* in nature. It is now well-known that the assumption of such a rational monopoly is questionable and unrealistic in real economies, since there is always an uncertainty and a time delay in collecting information and determining optimal responses, and in addition, function relations such as the market price function cannot be determined exactly based on theoretical consideration and observed data. Getting closer to the real world and improving the monopoly theory, we replace this extreme but conventional assumption with the more plausible one. Indeed, the monopolistic firm is assumed, first, to have only limited knowledge on the price function and, second, to obtain it with time delay. As a natural consequence

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of these alternations, the firm gropes for its optimal choice by using delay data obtained through market experiences. Such a boundedly rational monopoly model becomes *dynamic* in nature.

There are two major classes of dynamic economic models, depending on the selection of the time scales. If discrete time scales are assumed, then the monopoly may change output level at time periods $t = 1, 2, \ldots$ and time delays are also positive integers. In the recent literature with discrete-time framework, it has been demonstrated that a boundedly rational monopoly may exhibit simple as well as complex dynamic behavior. Nyarko [14] solves the profit maximizing problems without knowing the slope and intercept of a linear demand and shows that using Bayesian updating leads to cyclic actions and beliefs if the market demand is mis-specified. Furthermore, Puu [15] shows that the boundedly rational monopolist behaves in an erratic way under cubic demand with a reflection point. In the similar setting, Naimzada and Ricchiuti [13] represent that complex dynamics can arise even if cubic demand does not have a reflection point. Naimzada [12] exhibits that delay monopoly dynamics can be described by the well-known logistic equation when the firm takes a special learning scheme. More recently, Matsumoto and Szidarovszky [7] demonstrate that the monopoly equilibrium undergoes to complex dynamics through either a period-doubling or a Neimark–Sacker bifurcation.

Although it would not be difficult to insert time delays in a discrete-time setup, however increasing the number or lengths of time delays leads to a higher dimensional dynamic system that might be mathematically untractable. This study aims to consider the effects caused by different delay lengths on time evolutions of output and price when the firm uses the past price information obtained at different times. Thus it proposes monopoly dynamics in continuous time scale with the following three reasons and presents a new characterization of a monopoly's learning process under a limited knowledge of the market demand. First of all, the continuous-time model can approximate the discrete-time model if the time unit is small. Second, it gives a larger flexibility in the model in which output level changes might occur at any time and the delays can be any positive value. Lastly, it reveals that alternation of stability and instability occurs repeatedly in multiple delays framework, that is not found in discrete time framework.

This paper is a continuation of Matsumoto and Szidarovszky [11] (MS henceforth) where the monopolist does not know the price function and fixed time delays are introduced into the output adjustment process based on the gradient of the marginal expected profit. It also aims to complement Matsumoto and Szidarovszky [6,8] where uncertain delays are modeled by continuously distributed time delays when the firm wants to react to average past information instead of sudden market changes.[1] Under a circumstance in which price is uncertain and the price information is delayed, this paper examines the learning scheme in the cases of a single delay and two delays. It is an extended version of MS and thus has similarities and dissimilarities to MS. Its main purpose is to show that cyclic and erratic behavior can emerge from quite simple economic structures when uncertainty, information delays and behavioral nonlinearities are present is the same. Gradient dynamics without optimal behavior in MS is replaced with the learning scheme with profit maximizing behavior. In spite of this behavioral difference, derived mathematical equations and their solutions are the same. However the ways to arrive at the solutions are very different; an elementary method applicable only to the very special form of the mathematics equation is used in MS while we apply a more general method developed by Gu *et al.* [4]. As a result, the stability/instability conditions are simplified and clarified. Since economics behind the mathematical equations are different, the results to be obtained have different economic implications.

This paper develops as follows. The basic mathematical model is formulated and a single delay equation is examined in Section 2. In Section 3, it is assumed that the firm formulates its price expectation based on two delayed observations by using a delay feedback. Complete stability analysis is given, the stability regions are determined and illustrated. The occurrence of Hopf bifurcation is shown when one of the two delays is selected as a bifurcation parameter. The last section offers conclusions and further research directions.

## 2. The basic model

Consider a single product monopolist that sells its product to a homogeneous market. Let $q$ denote the output of the firm, $p(q) = a - bq$ the price function and $C(q) = cq$ the cost function.[2] Since $p(0) = a$ and $|\partial p(q)/\partial q| = b$, we call $a$ the *maximum price* and $b$ the *marginal price*. There are many ways to introduce uncertainty into this framework. In this study, it is assumed that the firm knows the marginal price but does not know the maximum price. In consequence it has only an estimate of it, which is denoted by $a^e$. So the firm believes that its profit is

$$\pi^e = (a^e - bq)q - cq$$

[1] There are two different ways to model time delays in continuous-time scale: fixed time delay and continuously distributed time delay (fixed delay and continuous delay henceforth). The choice of the type of delay results in the use of different analytical tools. In the cases of fixed delay, dynamics is described by a delay differential equation whose characteristic equation is a mixed polynomial-exponential equation with infinitely many eigenvalues. Bellman and Cooke [1] offer methodology of complete stability analysis in such models. On the other hand, in the cases of continuous delay, Volterra-type integro-differential equations are used to model the dynamics. The theory of continuous delays with applications in population dynamics is offered by Cushing [3]. Since Invernizzi and Medio [5] have introduced continuous delays into mathematical economics, its methodology is used in analyzing many economic dynamic models.

[2] Linear functions are assumed only for the sake of simplicity. We can obtain a similar learning process to be defined even if both functions are nonlinear. It is also assumed for the sake of simplicity that the firm has perfect knowledge of production technology (i.e., cost function).