Commun Nonlinear Sci Numer Simulat 28 (2015) 166–174

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Hidden attractor and homoclinic orbit in Lorenz-like system describing convective fluid motion in rotating cavity

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ARTICLE INFO

Article history:
Received 20 January 2015
Revised 6 March 2015
Accepted 7 April 2015
Available online 15 April 2015

Keywords:
Hidden attractor
Self-excited attractor
Multistability
Coexistence of attractors
Lorenz-like system
Homoclinic orbit
Lyapunov exponent
Lyapunov dimension

ABSTRACT

In this paper a Lorenz-like system, describing convective fluid motion in rotating cavity, is considered. It is shown numerically that this system, like the classical Lorenz system, possesses a homoclinic trajectory and a chaotic self-excited attractor. However, for the considered system, unlike the classical Lorenz system, along with self-excited attractor a hidden attractor can be localized. Analytical-numerical localization of hidden attractor is demonstrated.

1. Introduction

Consider the following physical problem: the convection of viscous incompressible fluid motion inside the ellipsoid

$$\left(\frac{x_1}{a_1}\right)^2 + \left(\frac{x_2}{a_2}\right)^2 + \left(\frac{x_3}{a_3}\right)^2 = 1, \quad a_1 > a_2 > a_3 > 0,$$

under the condition of stationary inhomogeneous external heating. It is assumed that the ellipsoid together with heat sources rotates with the constant velocity $\mathbf{\Omega_0}$ around its axis. Vector $\mathbf{l_0}$ determines the orientation of the ellipsoid in the space and has the same direction as the gravity vector $\mathbf{g}$. Vector $\mathbf{g}$ is stationary with respect to the ellipsoid motion. The value $\mathbf{\Omega_0}$ is assumed to be such that the centrifugal forces can be neglected in comparison with the influence of the gravitational field. Consider the case when the ellipsoid rotates around the axis x_3 that has a constant angle α with the gravity vector $\mathbf{g}$ ($|\mathbf{g}| = g$) and the vector $\mathbf{g}$ is placed in the plane x_1x_3. Then $\mathbf{\Omega_0} = (0, 0, \Omega_0)$ and $\mathbf{l_0} = (a_1 \sin\alpha, 0, -a_3 \cos\alpha)$. Let the steady-state temperature difference $\Delta\hat{\mathbf{T}} = (q_0,\ 0,\ 0)$ be generated along the axis x_1 (Fig. 1). Corresponding mathematical model (three-mode model of convection)

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http://dx.doi.org/10.1016/j.cnsns.2015.04.007
1007-5704/

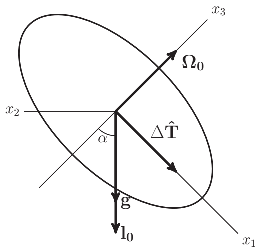

Fig. 1. Illustration of the problem setting.

was obtained by Glukhovsky and Dolzhansky [1] in the following form

$$\begin{cases} \dot{x} = Ayz + Cz - \sigma x, \\ \dot{y} = -xz + R_a - y, \\ \dot{z} = xy - z. \end{cases} \tag{1}$$

Here

$$\sigma = \frac{\lambda}{\mu}, \quad T_a = \frac{\Omega_0^2}{\lambda^2}, \quad R_a = \frac{g\beta a_3 q_0}{2a_1 a_2 \lambda\mu},$$
$$A = \frac{a_1^2 - a_2^2}{a_1^2 + a_2^2}\cos^2\alpha\, T_a^{-1}, \quad C = \frac{2a_1^2 a_2}{a_3(a_1^2 + a_2^2)}\sigma \sin\alpha,$$
$$x(t) = \mu^{-1}\left(\omega_3(t) + \frac{g\beta a_3 \cos\alpha}{2a_1 a_2 \Omega_0} q_3(t)\right), \quad y(t) = \frac{g\beta a_3}{2a_1 a_2 \lambda\mu} q_1(t),$$
$$z(t) = \frac{g\beta a_3}{2a_1 a_2 \lambda\mu} q_2(t),$$

and λ, μ, β are the coefficients of viscosity, heat conduction, and volume expansion, respectively; $q_1(t)$, $q_2(t)$, and $q_3(t)$ ($q_3(t) \equiv 0$) are temperature differences on the principal axes of ellipsoid a_1, a_2, and a_3, respectively; $\omega_1(t)$, $\omega_2(t)$, and $\omega_3(t)$ are the projections of the vectors of fluid angular velocities on the axes x_1, x_2, and x_3, respectively. Here

$$\omega_1(t) = -\frac{g\beta a_3}{2a_1 a_2 \Omega_0}\cos\alpha\, q_1(t), \quad \omega_2(t) = -\frac{g\beta a_3}{2a_1 a_2 \Omega_0}\cos\alpha\, q_2(t).$$

The parameters σ, T_a, and R_a are the Prandtl, Taylor, and Rayleigh numbers, respectively.

After the linear transformation (see, e.g., [1,2]):

$$x \to x, \quad y \to R - \frac{\sigma}{a_0 R + 1} C^{-1} z, \quad z \to \frac{\sigma}{a_0 R + 1} C^{-1} y,$$

one obtains the following system

$$\begin{cases} \dot{x} = -\sigma(x - y) - ayz \\ \dot{y} = rx - y - xz \\ \dot{z} = -z + xy, \end{cases} \tag{2}$$

where $a_0 = A/C^2$, $R = R_a C$,

$$a = \frac{a_0 \sigma^2}{(a_0 R + 1)^2}, \quad r = \frac{R}{\sigma}(a_0 R + 1). \tag{3}$$

System (2) with $a = 0$ coincides with the classical Lorenz system [3] with $b = 1$. As it is discussed in [2], system (2) can also be used to describe the following physical processes: waves interaction in plasma [4–7], the convective fluid motion inside rotating ellipsoid [1], the rotation of rigid body in viscous fluid [8], the gyrostat dynamics [9,10], the convection of horizontal layer of fluid making harmonic oscillations [11], and the model of Kolmogorov's flow [12].

Note that the Glukhovsky–Dolzhansky system is sufficiently different from the classical Lorenz system. In the Lorenz system, the convective fluid motion in two dimensions is considered only. In the Glukhovsky–Dolzhansky system, the convective fluid motion in three dimensions is considered which can be interpreted as one of the models of ocean flow [1].

In [13] for system (2) in the case $\sigma = \pm ar$ a detailed analysis of the equilibria stability and asymptotic behavior of trajectories is given and the values of parameters are obtained for which system (2) is integrable.

In what follows system (2) will be considered under the condition that the parameter a is positive. In this case if $r < 1$, then (2) has a unique equilibrium $S_0 = (0, 0, 0)$, which is globally asymptotically Lyapunov stable [2,14]. If $r > 1$, then system (2) has three equilibria: $S_0 = (0, 0, 0)$ and

$$S_{1,2} = (\pm x_1, \pm y_1, z_1). \tag{4}$$