Commun Nonlinear Sci Numer Simulat 28 (2015) 175–193

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

A Mellin transform approach to wavelet analysis

Gioacchino Alotta [a,*], Mario Di Paola [a], Giuseppe Failla [b]

[a] Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale e dei Materiali (DICAM), Università di Palermo, Viale delle Scienze, Ed. 8, 90128 Palermo, Italy

[b] Dipartimento di Ingegneria Civile, dell'Energia, dell'Ambiente e dei Materiali (DICEAM), Università Mediterranea di Reggio Calabria, Via Graziella, Località Feo di Vito, 89124 Reggio Calabria, Italy

ARTICLE INFO

Article history:
Received 6 December 2014
Revised 9 March 2015
Accepted 1 April 2015
Available online 15 April 2015

PACS:
02.30.Hq
02.30.Vv

Keywords:
Mellin transform
Fractional calculus
Wavelet analysis
Linear systems

ABSTRACT

The paper proposes a fractional calculus approach to continuous wavelet analysis. Upon introducing a Mellin transform expression of the mother wavelet, it is shown that the wavelet transform of an arbitrary function $f(t)$ can be given a fractional representation involving a suitable number of Riesz integrals of $f(t)$, and corresponding fractional moments of the mother wavelet. This result serves as a basis for an original approach to wavelet analysis of linear systems under arbitrary excitations. In particular, using the proposed fractional representation for the wavelet transform of the excitation, it is found that the wavelet transform of the response can readily be computed by a Mellin transform expression, with fractional moments obtained from a set of algebraic equations whose coefficient matrix applies for any scale a of the wavelet transform. Robustness and computationally efficiency of the proposed approach are shown in the paper.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

The key concept of the wavelet transform is to provide a time-frequency representation of a signal based on a double series of functions called "wavelets" [1–3]. A wavelet is a local function of time, generated by scaling and shifting a single "mother" function. By scaling, the time duration of the wavelet can be adjusted according to the local frequency content of the signal while, by shifting, the time evolution of the frequency content can be investigated. Wavelets at small scales provide an accurate time resolution, to capture high-frequency components associated with short-lived phenomena, and wavelets at large scales provide an enhanced frequency resolution to capture low-frequency components representing long-lasting trends. Wavelet analysis is now a well-established tool for time-frequency analysis of many signals, with significant advantages over standard Fourier transform and short-time Fourier transform [4]. Wavelet families associated with specific mother functions have proved appropriate for a variety of mechanics related problems as, among others, analyses of dynamic systems with time-varying characteristics [4], analyses of structural responses to non-stationary excitations [5,6], time-varying spectra estimation [7–10], system identification [11,12], damage detection [13,14], solutions of boundary-value problems [15] and functional equations [16,17], finite element analysis [18]. Continuous and discrete wavelet transforms exist. The discrete wavelet transform differs from the continuous one as involves a discrete set of scales and shifts, and is particularly suitable for digital signal processing. Comprehensive reviews on the use of continuous and discrete wavelet transforms may be found in refs. [18–20].

* Corresponding author. Tel.: +393290264959.
E-mail address: gioacchino.alotta@unipa.it (G. Alotta).

http://dx.doi.org/10.1016/j.cnsns.2015.04.001
1007-5704/© 2015 Elsevier B.V. All rights reserved.

Despite the voluminous body of literature, there are still several issues worth investigating in wavelet analysis, both theoretical and numerical. From a theoretical point of view, it may be of interest to investigate potential relations between the wavelet transform and alternative transforms available in the literature. From a numerical point of view, for instance, an efficient computation of the wavelet transform is highly desirable when performing wavelet analysis of the dynamic response of a system. An attempt to provide a response to these questions is pursued, in this paper, by fractional calculus concepts.

Fractional calculus has attracted a growing interest in the last decades [21–24]. Fractional operators have proved successful in representing long memory or non-local effects [25–28], with typical applications to viscoelasticity [29–33] and biomechanics [34]. The suitability of fractional operators for representing functions/variables of particular interest in engineering has been widely investigated, and applications have been proposed for probability density functions, characteristic functions, power spectral densities [35–37]. In particular, in this context a Mellin transform representation has been used, i.e. an integral representation involving fractional moments of the transformed function [35–37].

Relations between wavelet transform and fractional calculus have been investigated in several, very recent studies [38–47], where wavelets have been used to build numerical solutions of fractional differential equations with various fractional operators, or fractional wavelet families have been used [48,49]. A different perspective is taken in this paper because here, as in a few previous studies [50], the main purpose is to apply fractional calculus concepts to wavelet analysis. This objective is pursued in two steps. First, starting from a Mellin transform expression of the mother wavelet, the wavelet transform of an arbitrary function $f(t)$ is given a suitable fractional representation involving Riesz integrals of the transformed function $f(t)$, and corresponding fractional moments of the mother wavelet. Second, using the proposed fractional representation for the wavelet transform of the excitation of a linear system, it is shown that the wavelet transform of the response can be computed by a Mellin transform expression with fractional moments obtained from a set of algebraic equations, whose coefficient matrix applies for any scale a of the wavelet transform. Computational advantages of the proposed approach are discussed throughout the paper, while its robustness is substantiated by numerical applications.

The paper comprises five sections. Wavelet transform and Mellin transform are discussed in Section 2. Sections 3 and 4 present respectively the proposed fractional representation of the wavelet transform and the Mellin transform approach to wavelet analysis of linear systems under arbitrary excitations. Numerical applications are reported in Section 5.

2. Theoretical background

This Section illustrates basic concepts of wavelet transform and Mellin transform. Attention is focused on some properties of the Mellin transform, which will be used to formulate the fractional calculus approach to wavelet analysis proposed in the paper.

2.1. Wavelet transform

The continuous wavelet transform of a function $f(t)$ is defined as [1,2]:

$$W_f(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right)dt \tag{1}$$

where $\psi(t)$ is the mother wavelet, $a \in \mathbb{R}^+$ and $b \in \mathbb{R}$ are scale and shift of the mother wavelet along the t domain. Both a and b are continuous parameters within the respective domain. Time and frequency localization properties of the wavelet transform depend on the value of the scale a. As a approaches zero or $+\infty$, the compressed or dilated wavelet $a^{-1/2}\psi((t-b)/a)$ provides increasingly sharper or coarser time resolution, with the corresponding wavelet transform $W_f(a,b)$ displaying the small-scale/high-frequency or large-scale/low-frequency features of the function $f(t)$, at various locations b.

From the set of coefficients $W_f(a,b)$ the function $f(t)$ can be reconstructed in the form:

$$f(t) = \frac{1}{\pi c_\psi}\int_{-\infty}^{\infty}\int_0^{\infty}\frac{W_f(a,b)}{a^{3/2}}\psi\left(\frac{t-b}{a}\right)dadb, \quad c_\psi = \int_{-\infty}^{\infty}\frac{|\Psi(\omega)|^2}{|\omega|}d\omega < \infty \tag{2a,b}$$

where $\Psi(\omega)$ is the Fourier transform of the mother wavelet $\psi(t)$. Notice that Eq. (2b) corresponds to a certain number of sub-conditions on the mother wavelet $\psi(t)$, as detailed in ref. [19].

2.2. Mellin transform

Be $f(t) \in \mathbb{R}$ a square-integrable function defined over the $t \geq 0$ domain. The Mellin transform of complex order $\gamma = \rho + i\eta$ and the corresponding inverse are defined as:

$$F_M(\gamma) = M\{f(t);\gamma\} \equiv \int_0^{\infty} t^{\gamma-1}f(t)dt \tag{3}$$

$$f(t) = M^{-1}\{F_M(\gamma);t\} \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty}F_M(\gamma)t^{-\gamma}d\eta \quad (t>0) \tag{4}$$

For Eqs. (3) and (4) to hold, $\rho = \mathrm{Re}[\gamma]$ must belong to the so-called *fundamental strip* of the complex plane $-p < \rho < -q$. The limits of the fundamental strip are related to the asymptotic behavior of the transformed function $f(t)$, i.e. $f(t) \sim t^p$ for $t \to 0$ and $f(t) \sim t^q$ for $t \to \infty$. Notice that the Mellin transform (3) can also be referred to as complex fractional moment of order $(\gamma-1)$ of the function $f(t)$ [37].

Download English Version:

https://daneshyari.com/en/article/758212

Download Persian Version:

https://daneshyari.com/article/758212

Daneshyari.com