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Adaptive cluster general projective synchronization of complex dynamic networks in finite time

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ABSTRACT

This paper investigates adaptive cluster general projective synchronization (CGPS) of complex dynamic networks in finite time. Based on the finite time synchronization control techniques and Lyapunov stability theorem, sufficient conditions are derived to guarantee the realization of adaptive cluster general projective synchronization. Finally, numerical simulation is provided to support the theoretical results.

1. Introduction

The number of studies on synchronization of complex networks have dramatically increased during the past years. One of the most significant reasons lies in its potential applications in areas such as biology system, physics, communication and traffic [1–4]. As a result, different types of synchronization protocols have been put forward and deeply studied, for example, complete synchronization, phase synchronization, cluster synchronization, etc. [5–8]. These protocols are all meaningful in the procedure of studying the synchronization categories. However, the dynamical node of a real-world complex network is too large to study the whole characters. Thus, the cluster synchronization with the aim to study partial characteristics of complex network has been receiving mounting attentions [9–12].

Cluster synchronization mainly focuses on the synchronization between drive system and the coupled oscillators split into subgroups called cluster. Until now, a large variety of works with various cluster synchronization protocols have been presented. Zhang and Ma et al. studied the problems of cluster synchronization, which is induced by one-node cluster in networks with asymmetric negative couplings [13]. Wang and Cao in the same year, published their researches on cluster synchronization in nonlinearly coupled delayed networks of non-identical dynamic systems [14]. In addition, Li and Wang et al. investigated the pinning cluster synchronization for delayed dynamical networks via Kronecker product [15]. As an extension in complex

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networks, the work [16] discussed cluster synchronization in colored community network with different order node dynamics. Especially, Cai and Jiang et al. have also expanded the cluster synchronization from general networks to overlapping networks [17] with time-varying impulses. To sum up, these literatures contribute an important step on cluster synchronization. However, their obtained results are all based on construction of general error. Although the authors [10] have considered the projective synchronization in cluster synchronization, its results on cluster synchronization cannot extend to general projective synchronization. Therefore, we especially pay attention to this problem and present the researches on adaptive cluster general projective synchronization. With our best knowledge, this scheme has never been reported before, which motivates the current work.

On the other hand, the synchronization time is also an important factor in terms of control cost. Due to this fact, most of the researchers were concerned with asymptotical or exponential synchronization of networks through impulsive control, intermittent control or hybrid control to reduce the synchronization time [18–20]. The obtained results show the effective ability to accelerate the synchronization speed. But there also remain an important challenge about how to estimate the synchronization time. Due to this fact, an effective method of finite time synchronization control technique has been put forward [13–21]. This measure is effective to compute the synchronization time. But the existing problem is that few works are involved in cluster synchronization of complex networks in finite time, which is the second motivation for current work.

Inspired by the above discussions, this paper mainly concentrates on the two important proposed problems, and investigates the adaptive cluster general projective synchronization in finite time. Detailed criteria are obtained through strict theoretical analysis. The numerical example at last is given to show the effectiveness of the synchronization criteria. The most important highlights are: (1) the definition of cluster general projective synchronization (CGPS) is firstly given, and then detail analysis of this type of cluster synchronization of two general networks are discussed; (2) Through finite time synchronization technique, we obtain the finite time scope of cluster synchronization; (3) sufficient conditions for achieving CGPS are derived and numerical simulation verifies the obtained results.

This paper is organized as follows. Section 2 expresses prepared knowledge and problem statement. In Section 3, assumption, lemma and the definition of cluster general projective synchronization (CGPS) are given. Lyapunov stability theory is carried out to prove the proposed theories in Section 4. Example and simulation are given in Section 5. Finally, conclusions are derived in Section 6.

2. Problem statement

In this paper, we consider a complex network consisting of N linearly and diffusively coupled nodes described as follows:

$$\dot{x}_i(t) = F_i(t, x_i(t), \alpha_i) + \sum_{j=1}^{N} c_{ij} T x_j(t), \quad i \in \widehat{U} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^{\mathrm{T}} \in R^n$ is the state vector of the ith node at moment t; $\widehat{U} = \{k_{l-1}+1, \dots, k_l\}$ denotes the index set of all the nodes in the kth cluster, $k = 1, 2, \dots, m, m_l = N, l_{k-1} < l_k$; $F_i(t, x_i(t), \alpha_i)$ indicates the ith node dynamics, it also can be expressed as the forms: $F_i(t, x_i(t), \alpha_i) = f_i(t, x_i(t)) + g_i(t, x_i(t)) \cdot \alpha_i$, where $f_i(\cdot)$ and $g_i(t, x_i(t)) : R^n \to R^n$ are nonlinear vector-valued functions, which are distinct for different cluster, representing the activity of an individual subsystem; The matrix $C = (c_{ij})_{N \times N}$ is the zero-row-sum outer-coupling matrix. If there is a connection from node i to node j $(i \neq j)$, then $c_{ij} \neq 0$, otherwise, $c_{ij} = 0$. T represents the inner coupling matrix, and is assumed to be $T = vI_{n \times n}$, where v is a constant.

Remark 1. The outer coupling matrix $C = (c_{ij})_{n \times n}$ not only satisfies the rules in which if there is a link coupling from node i to node j $(i \neq j)$, then $c_{ij} = 1$ [10], but can also be bounded constants if there is a link coupling from node i to node j $(i \neq j)$.

3. Description of the identification scheme

In this section, we will present a general response network consisting of N nodes regarding the networks (1) as the drive network:

$$\dot{y}_i(t) = F_i(t, y_i(t), \alpha_i) + \sum_{j=1}^{N} c_{ij} T y_j(t) + U_i \quad i \in \widehat{U} \tag{2}$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \dots, y_{in}(t))^{\mathrm{T}} \in R^n$ is the state vector of the ith node; $f_i(t, y_i(t))$ and $g_i(t, y_i(t)) : R^n \to R^n$ are also nonlinear vector-valued functions, which are distinct for each cluster, representing the activity of an individual subsystem; U_i is an outer controller.

Before giving the main results, we will introduce definition, assumption, lemma which are required in this paper.

Definition 1. Given a vector function $\varphi : R^n \to R^n$, if the complex networks (1) and (2) satisfies $\lim_{t\to\infty} \|y_{\widehat{U}}(t) - \varphi(x_{\widehat{U}}(t))\| = 0$, where $\widehat{U}$ represents the index set of all the nodes in the kth cluster $(k = 1, 2, \dots, m)$, and $\varphi(x)$ is differentiable at moment t, we then call that the networks (1) and (2) can achieve cluster general projective synchronization (CGPS).

Assumption 1. If the vector-valued function $F(t, x(t), \alpha)$ satisfies the Lipschitz condition, there exists a constant $L > 0$ such that for $\forall x(t), y(t) \in R^n$, we have

$$\|F(t, y(t), \alpha) - F(t, x(t), \alpha)\| \le L \|y(t) - x(t)\|. \tag{3}$$