Commun Nonlinear Sci Numer Simulat 28 (2015) 201–209

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Secondary whirls in thermoconvective vortices developed in a cylindrical annulus locally heated from below

D. Castaño, M.C. Navarro*, H. Herrero

Departamento de Matemáticas, Facultad de Ciencias y Tecnologías Químicas, Universidad de Castilla-La Mancha, 13071 Ciudad Real, Spain

ARTICLE INFO

Article history:
Received 3 July 2014
Revised 13 March 2015
Accepted 21 April 2015
Available online 29 April 2015

Keywords:
Thermoconvective instabilities
Vortices
Navier–Stokes equations
Spectral methods

ABSTRACT

This paper shows numerically the formation of secondary whirls embedded in axisymmetric vertical vortices generated in a cylindrical annulus non-homogeneously heated from below. This secondary circulation, that is formed near the center of the primary vortex, appears after a thermoconvective instability. The transition from the axisymmetric vortex to the time-dependent flow where the subvortex is found is studied using nonlinear simulations. The size of the inner radius of the cylindrical annulus is relevant for the appearance or not of subvortices. They only form for small and medium values of the inner radius. The temperature profile at the bottom affects the intensity of the subvortex generated. Results are remarkable as they qualitatively describe observations in dust devils.

1. Introduction

Dust devils are columnar, ground-based whirlwinds, common in dry regions, observed in daytime and made by the dust picked up from the ground [1,2]. They are formed when surface isolation leads to a superadiabatic lapse rate, causing an unstable stratified atmosphere and strong convection [2,3]. They have a vortical structure, characterized by a spiral up motion around an eye. Dust devils frequently contain subvortices, small “parasites” or secondary circulation embedded in the primary whirl (dust devil) that normally form near the center of the dust devil and follow essentially concentric circular paths about the dust devil center [3–7].

On simulating dust devils, most of the experiments either laboratory or numerical impose artificially a rotating fluid [8,9]. In Ref. [10] authors show numerically the relevance of convective mechanisms in the generation of axisymmetric vertical vortices, qualitatively similar to dust devils, in a cylindrical annulus and prove that this primary vortex can appear spontaneously after a thermal bifurcation of a stationary axisymmetric nonrotating convective flow, when two temperature gradients (vertical and horizontal) come into play. That work includes a study of the region of parameters where the vortex is formed and a stability analysis of its structure. In Ref. [11] authors analyze the influence of the size of the radius of the inner cylinder on the structure, intensity and stability of the primary vortex developed. In Ref. [12] it is studied the effect of the shape of the temperature profile at the bottom.

In the present work we show that a thermoconvective instability is responsible for the formation of secondary whirls embedded in the primary axisymmetric vortex and study the influence of the inner radius and the sharpness of the temperature profile at the bottom on these parasitic whirls.

* Corresponding author. Tel.: +34 926 295300x3469; fax: +34 926 29 5318.
E-mail address: MariaCruz.Navarro@uclm.es (M.C. Navarro).

http://dx.doi.org/10.1016/j.cnsns.2015.04.019

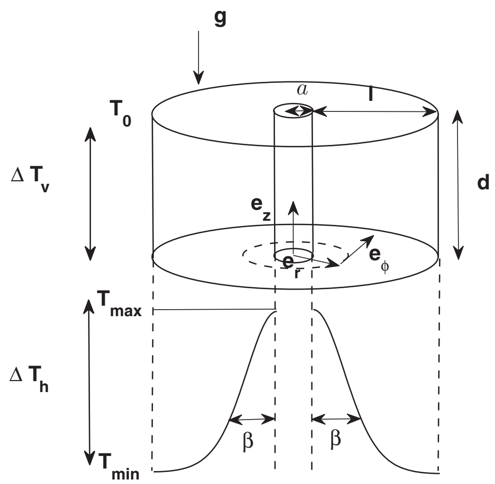

Fig. 1. Physical setup.

The paper is organized as follows. Section 2 presents the physical setup and the mathematical formulation of the problem in a dimensionless form. Section 3 describes the numerical implementation. Section 4 presents the numerical results on the formation of secondary whirls, and the analysis of the influence that the radius of the inner cylinder and the profile of the temperature at the bottom have on the generation and structure of the secondary circulation. Finally, in Section 5, conclusions are presented.

2. Formulation of the problem

The physical setup (see Fig. 1) consists of a horizontal fluid layer in a container bounded by two concentric cylinders of radii a and $a + l$ (r coordinate) as in Ref. [10]. The value of a will be varied in the study but keeping fixed the value of $a + l$. The depth of the domain is d (z coordinate) and it is also kept fixed. Therefore, the dimensions of the outer cylinder do not change. At $z = 0$ the imposed temperature has a Gaussian profile which takes the value $T_{\max}$ at $r = a$ and the value $T_{\min}$ at the outer part ($r = a + l$). The upper surface is at temperature $T = T_0$. We define $\triangle T_v = T_{\max} - T_0$, $\triangle T_h = T_{\max} - T_{\min}$ and $\delta = \triangle T_h / \triangle T_v$.

In the governing equations, $\mathbf{u} = (u_r, u_\phi, u_z)$ is the velocity field, T is the temperature, p is the pressure, r is the radial coordinate, and t is the time. They are expressed in dimensionless form after rescaling: $\mathbf{r}' = \mathbf{r}/d$, $t' = \kappa t/d^2$, $\mathbf{u}' = d\mathbf{u}/\kappa$, $p' = d^2 p/(\rho_0 \kappa \nu)$, $\Theta = (T - T_0)/\triangle T_v$. Here $\mathbf{r}$ is the position vector, κ the thermal diffusivity, ν the kinematic viscosity of the liquid, and ρ_0 the mean density at temperature T_0. The domain is $\mathcal{D} = [\bar{a}, \bar{a} + \Gamma] \times [0, 1] \times [0, 2\pi]$ where $\Gamma = l/d$ and $\bar{a} = a/d$.

The system evolves according to the momentum, mass balance and energy conservation equations, which in dimensionless form (with primes now omitted) are,

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\partial_t \Theta + \mathbf{u} \cdot \nabla \Theta = \nabla^2 \Theta, \tag{2}$$

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\, \mathbf{u} = Pr(-\nabla p + \nabla^2 \mathbf{u} + R\Theta \mathbf{e}_z), \tag{3}$$

where the operators and fields are expressed in cylindrical coordinates and the Oberbeck–Boussinesq approximation has been used. Here $\mathbf{e}_z$ is the unit vector in the z direction. The following dimensionless numbers have been introduced: the Prandtl number $Pr = \nu/\kappa$, and the Rayleigh number $R = g\alpha \triangle T d^3 / \kappa \nu$, which represents the effect of buoyancy and in which α is the thermal expansion coefficient and g the gravitational acceleration.

Regarding boundary conditions, at $r = \bar{a}$, the lateral wall of the inner cylinder, the velocity is zero and an insulating wall is considered,

$$u_r = u_\phi = u_z = \partial_r \Theta = 0, \quad \text{on} \quad r = \bar{a}. \tag{4}$$

At $r = \bar{a} + \Gamma$, we establish that the fields do not change radially, a lateral inflow/outflow is permitted and an insulating wall is considered,

$$\partial_r u_r = \partial_r u_\phi = \partial_r u_z = \partial_r \Theta = 0, \quad \text{on} \quad r = \bar{a} + \Gamma. \tag{5}$$

On the top surface no-slip boundary conditions are considered, assuming that the top boundary is located where there is no motion yet, therefore the velocity is zero and the temperature is $T = T_0$, that after rescaling become,

$$u_r = u_\phi = u_z = \Theta = 0, \quad \text{on} \quad z = 1, \tag{6}$$