Commun Nonlinear Sci Numer Simulat 19 (2014) 2559–2567

Contents lists available at SciVerse ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Short communication

Optimal control of a parabolic distributed parameter system via radial basis functions

J.A. Rad [a], S. Kazem [b], K. Parand [c,*]

[a] Department of Mathematics, Faculty of Mathematical Sciences, Shahid Beheshti University, Evin, Tehran 19839, Iran
[b] Department of Applied Mathematics, Faculty of Mathematics and Computer Science, Amirkabir University of Technology, No. 424, Hafez Ave., Tehran, Iran
[c] Department of Computer Sciences, Faculty of Mathematical Sciences, Shahid Beheshti University, Evin, Tehran 19839, Iran

ARTICLE INFO

Article history:
Received 20 August 2011
Received in revised form 8 January 2013
Accepted 9 January 2013
Available online 19 February 2013

Keywords:
Optimal control
Distributed parameter systems
Radial basis functions
Collocation method
Gaussian RBF
Lagrange multipliers

ABSTRACT

This paper attempts to present a meshless method to find the optimal control of a parabolic distributed parameter system with a quadratic cost functional. The method is based on radial basis functions to approximate the solution of the optimal control problem using collocation method. In this regard, different applications of RBFs are used. To this end, the numerical solutions are obtained without any mesh generation into the domain of the problems. The proposed technique is easy to implement, efficient and yields accurate results. Numerical examples are included and a comparison is made with an existing result.

1. Introduction

Optimal control is widely applied in aerospace, engineering, economics and other areas of science and has received considerable attention of researchers. Up to now, enormous effort has been spent on the development of computational methods for generating solutions of optimal control problems [1–12]. Although many computational methods have been developed and proposed, the modification of the existing methods and development of new method should yet be explored to obtain accurate solutions successfully. One of the most important methods is the spectral method which has received considerable attention in dealing with various problems of dynamic systems.

Consider the one-dimensional diffusion equation

$$\frac{\partial x(y,t)}{\partial t} = \frac{\partial^2 x(y,t)}{\partial y^2} + u(y,t), \tag{1.1}$$

with the initial condition

$$x(y,0) = f(y), \quad 0 \leqslant y \leqslant L \tag{1.2}$$

and the boundary conditions

* Corresponding author.
E-mail addresses: j.amanirad@gmail.com, j_amanirad@sbu.ac.ir (J.A. Rad), saeedkazem@gmail.com (S. Kazem), k_parand@sbu.ac.ir (K. Parand).

1007-5704/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.cnsns.2013.01.007

$$\frac{\partial x(y,t)}{\partial y}=0,\quad \text{at } y=0,\ 0\leqslant t\leqslant t_f,$$
$$\frac{\partial x(y,t)}{\partial y}=0,\quad \text{at } y=L,\ 0\leqslant t\leqslant t_f. \tag{1.3}$$

The purpose is to find the optimal control $u(y,t)$ that minimizes the following cost functional:

$$J=\frac{1}{2}\int_0^{t_f}\int_0^{L}\left(Q'x^2(y,t)+R'u^2(y,t)\right)dydt, \tag{1.4}$$

where Q' and R' are non-negative, positive weighting values, respectively.

The numerical methods used to find the optimal control of parabolic distributed parameter systems has been presented by [13–17]. Sage and White [13] used a finite difference technique, Mahapatra [14] derived a piecewise continuous solution using Walsh functions and Wang and Chang [15] transformed the optimal control problem into a two-point boundary value problem and obtained the solution using shifted Legendre polynomials. Horng and Chou [16] reduced the optimal control of a distributed parameter system into the optimal control of a linear time-invariant lumped parameter system; furthermore, they derived the integral of the cross-product of two shifted Chebyshev vectors to find the solution. Chang and Yang [17] transformed the optimal control problem into a two point boundary value problem; they also derived the operational matrix for the integration of the generalized orthogonal polynomials and obtained the optimal control using the Taylor series and several kinds of orthogonal polynomials, by employing only the cross-product of two shifted Legendre vectors. Razzaghi and Arabshahi [18] transformed the optimal control problem into a two point boundary value problem and adopted an approach using the Taylor series. For more references, see [19,20].

The interpolation of a given set of points is an important problem especially in higher dimensional domains. Although polynomials (e.g. Chebyshev and Legendre) are very powerful tools for interpolating of a set of points in one-dimensional domains, the use of these functions are not efficient in higher dimensional or irregular domains. While applying these functions, the points in the domain of the problem should be chosen in a special form, which is very limiting when the interpolation of a scattered set of points is needed. Radial basis functions (RBFs) are very efficient instruments for interpolating a scattered set of points, which have been used in the last 20 years. The use of radial basis functions collocation method for solving partial differential equations has some advantages over mesh dependent methods, such as finite-difference methods, finite element methods, spectral elements, finite volume methods and boundary element methods. Since a large portion of the computational time is spent for providing a suitable mesh on the domain of the problem in mesh-dependent methods, the meshfree methods have an auxiliary role in the numerical solution of partial differential equations. In recent years, to avoid the mesh generation, meshfree methods have attracted the attention of researchers. In the literature, several meshfree methods have been presented, such as the smooth particle hydrodynamics (SPH) [21], the diffuse element method (DEM) [22], the partition of unity method (PUM) [23], the hp-cloud method [24], the element free Galerkin (EFG) method [25], the reproducing kernel particle method (RKPM) [26], the meshfree collocation method based on radial basis functions (RBFs) [27–30], the meshfree local Petrov–Galerkin (MLPG) method [31–33], the radial point interpolation method [34] and so on. For a review of meshfree methods, the interested reader is referred to [34].

RBFs were first studied by Roland Hardy, an Iowa State geodesist, in 1968. This method allows scattered data to be easily used in computations. An extensive study of interpolation methods available at the time was conducted by Franke [35], and concluded that RBFs interpolations were evaluated as the most accurate techniques. The theory of RBFs originated as a mean to prepare a smooth interpolation of a discrete set of data points. The concept of using RBFs for solving differential equations (DEs) was first introduced by Kansa [27,36], who directly collocated the radial basis functions for the approximate solution of DEs. However, in recent years, RBFs have been extensively researched and applied in a wider range of analysis. Partial differential equations (PDEs) and ordinary differential equations (ODEs) have been solved using RBFs with recent work [28–30,37–45].

A well-known space RBF $\Phi(\|X-X_i\|):\mathbb{R}^+\to\mathbb{R}$ depends on the separation of a field point $X\in\mathbb{R}^d$ and the data centers X_i, for $i=1,2,\ldots,N$, and N data points. The interpolants are classed as radial function due to their spherical symmetry around centres X_i, where $\|.\|$ is the well-known Euclidean norm. The most known space RBFs are listed in Table 1, where $r=\|X-X_i\|$ and ϵ is a free positive parameter, often referred to as the shape parameter, to be specified by the user. The shape parameter ϵ within the Gaussian and multiquadric RBFs requires fine tuning and can dramatically alter the quality of the interpolation. Too large or too small shape parameter ϵ makes the GA too flat or too peaked. Despite many research works, which are done to find algorithms for selecting the optimum values of ϵ [46–50], the optimal choice of shape parameter is an open problem, which is still under intensive investigation.

One of the most powerful interpolation methods with analytic d-dimensional test function is the space RBFs method, based on Gaussian (GA) basis function

$$\phi(r)=e^{-\epsilon^2 r^2},\quad \epsilon>0.$$

In the cases of inverse quadratic, inverse multiquadric (IMQ) and Gaussian (GA), the coefficient matrix of RBFs interpolating is positive definite and, for multiquadric (MQ), it has one positive eigenvalue and the remaining ones are all negative [51].