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Review

Bifurcations of phase portraits of a Singular Nonlinear Equation of the Second Class

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ABSTRACT

The soliton dynamics is studied using the Frenkel Kontorova (FK) model with non-convex interparticle interactions immersed in a parameterized on-site substrate potential. The case of a deformable substrate potential allows theoretical adaptation of the model to various physical situations. Non-convex interactions in lattice systems lead to a number of interesting phenomena that cannot be produced with linear coupling alone. In the continuum limit for such a model, the particles are governed by a Singular Nonlinear Equation of the Second Class. The dynamical behavior of traveling wave solutions is studied by using the theory of bifurcations of dynamical systems. Under different parametric situations, we give various sufficient conditions leading to the existence of propagating wave solutions or dislocation threshold, highlighting namely that the deformability of the substrate potential plays only a minor role.

Contents

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1. Introduction

Simplicity and universality are two key requirements of a good physical model. Universal models, which can be used to describe a variety of different phenomena, are very rare and yet are of major importance. They have been proved useful at several scales and moreover possess an educational value. A simple example of such models is the Frenkel Kontorova (FK) model [1]. The FK model has become very popular in many niches of solid state and nonlinear physics. They invented their model in order to describe the motion of a dislocation in crystals. Meanwhile, the FK model has also become a model for an adsorbate layer on the surface of crystal, for ionic conductors, for glassy materials, for charge-density-wave (CDW) transport, and for chains of coupled Josephson junctions. Especially, a number of current researches, such as sliding friction [2,3], heat conduction [4–6], chaos control and nonlinear coupled pendulum [7] are also carried out using the FK model.

The standard FK model describes a chain of coupled atoms subjected to an external periodic potential and its Hamiltonian is defined as

$$H = T + U. \tag{1}$$

T and U are the kinetic and potential energies respectively. U consists of interparticle interactions V_c that take into account the linear and nonlinear couplings between the nearest neighbors of the chain and the substrate external potential V_e along the chain with spatial period a. b stands for the natural spacing of the unperturbed chain. The contribution of the particle i to potential energy U is then defined as follows:

$$U(x_i) = V_c(r_i) + V_e(x_i), \quad 1 \leqslant i \leqslant n, \tag{2}$$

where $r_i = x_{i+1} - x_i - b + a$ represents the distance between the nearest neighbors.

One of the basic restrictions of this traditional model adopted originally by the authors, which makes it applicable for small lattice misfits only, is the purely elastic interaction between neighboring atoms as a substitute for the real inter-atomic forces. In addition, in real physical systems, the shape of the substrate potential can deviate from the standard (sinusoidal or rigid) one and this may affect strongly the transport properties of the system.

As a better approach for modeling real systems, many researchers have considered the FK model while either modifying the external periodic potential [1] or modifying interactions between atoms [8–12]. For realistic anharmonic interactions, such as those of Toda, Morse, Markov and Trayanov, besides the position of the minimum there exists a second characteristic length at which the tensile strength of the bond reaches its maximum, that is the inflection point r_{inf}. This makes the behavior of such systems more complex. The mathematical challenges of solving the problem analytically increase since the equations are no more single valued. It has been reported in a numerical study of the discrete FK model with anharmonic interactions (Toda, Morse) [13] that beyond some critical values of the independent parameters (the natural lattice mismatch and/or amplitude of the external periodic potential), these equations have no solution which was interpreted as a disintegration of the system.

This work aims to consider a non-convex pair potential V_c [13–15] defined as:

$$V_c(r) = \frac{V_0 B\left(\frac{r}{r_0}\right)^2}{1 + B\left(\frac{r}{r_0}\right)^2}, \tag{3}$$

with $r_0 = \frac{a}{2\pi} \cdot B$ changes the width of the potential well with depth V_0. It is a single non-convex even potential with a single minimum and infinitely differentiable. The curvature (elastic constant) of $V_c(r)$ is $2BV_0$ at $r = 0$, and $V_c(r)$ has inflection points at $\pm r_{inf} = \frac{r_0}{\sqrt{3B}}$. This potential, plotted versus the nearest neighbors distance $\frac{r}{r_0}$ in Fig. 1, is infinitely differentiable

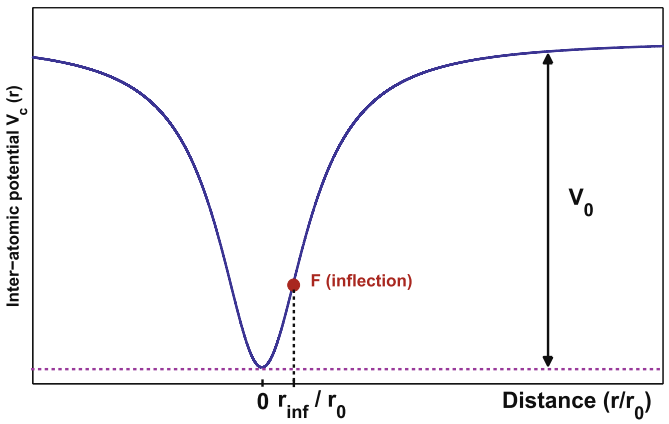

Fig. 1. Plot of the coupling potential $V_c(r)$.