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A good approximation of modulated amplitude waves in Bose–Einstein condensates

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ABSTRACT

In this paper, we present a perturbation method that utilizes Hamiltonian perturbation theory and averaging to analyze spatio-temporal structures in Gross–Pitaevskii equations and thereby investigate the dynamics of modulated amplitude waves (MAWs) in quasi-one-dimensional Bose–Einstein condensates with mean-field interactions. A good approximation for MAWs is obtained. We also explore dynamics of BECs with the nonresonant external potentials and scatter lengths varying periodically in detail using Hamiltonian perturbation theory and numerical simulations.

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1. Introduction

The dynamics of Bose–Einstein condensates (BECs) is a fundamental phenomenon connected to superfluidity and superconductivity in liquid helium [1]. In the mean-field approximation, and at sufficiently low temperatures, the dynamics of matter waves in BECs are accurately described by the Gross–Pitaevskii (GP) equation [2], namely a variant of the nonlinear Schrödinger (NLS) equation with an external potential and the nonlinearity coefficient

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + g(x)|\psi|^2\psi + V(x)\psi, \tag{1.1}$$

where ψ is the mean-field condensate wave function (with density $|\psi|^2$ measured in units of the peak 1D density n_0), x and t are normalized, respectively, to the healing length $\xi = \hbar/\sqrt{n_0|g_1|m}$ and ξ/c (where $c = \hbar\sqrt{n_0|g_1|/m}$ is the Bogoliubov speed of sound), and energy is measured in units of the chemical potential $\delta = g_1 n_0$. In the above expressions, $g_1 = 2\hbar\omega_\perp a_0$, where $\omega_\perp$ denotes the confining frequency in the transverse direction, and a_0 is a characteristic value of the scattering length. The nonlinearity coefficient $g(x)$ is caused by the spatially varying scattering length, which has been crucial to many experimental achievements, such as the formation of molecular condensates and probing the so-called BEC–BCS crossover. Of particular interest are BECs in optical lattices (periodic potentials), which have already been used to study Josephson effects, squeezed states, Landau–Zener tunneling and Bloch oscillations, and the transition between superfluidity and Mott insulation.

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Finding exact solutions to the GP equation in nontrivial settings is a problem of great significance [3], and the literature of the exact solutions for the GP equation' (1.1) consists of numerous papers, for some recent work we can refer to [4–8]. Modulated amplitude waves (MAWs) with the uniformly propagating coherent structures

$$\psi(t,x) = R(x)\exp(i[\Theta(x) - \mu t]), \tag{1.2}$$

as a class of exact solutions, yielding a special interest, which generalize the Bloch modes occurring in linear system with periodic potentials [9], have been extensively studied [10,11] in case of θ independent of x (standing waves), where $\psi(t,x)$ is periodic and quasiperiodic with respect to the spatial variable x. Meanwhile, the MAWs in the complex Ginzburg–Landau system have been studied by Brusch and co-authors [12].

The dynamics of solutions with coherent structure (1.2) for GP equation have been studied in recent paper [14,13] by the averaged method and topology degree theory. In these papers, a complicated transform is constructed, which can change a singular system into a standard form of averaging. However, as stated in [13], this transformation is not exact symplectic, and the Hamiltonian structure has been destroyed which maybe lead to loss of dynamical information of the original system. Therefore, looking for an appropriate symplectic transformation is an important topic for discussion.

In this paper, we will investigate the dynamics of MAWs by the averaged method and Hamiltonian perturbed method. We transform the original system with the Hamiltonian form into a simple one by constructing a series of canonical transformations via generating functions. The dynamics of the transformed system is easily studied. If the frequencies of the external potential $V(x)$ and the nonlinearity coefficient $g(x)$ are nonresonant to the ones of the unperturbed system, the existence of infinitely many periodic and quasiperiodic MAWs can be proved by the Moser twist theorem and Poincaré–Birkhoff twist theorem. In this situation, a good approximation of order one for MAWs is obtained. In the other case, we can only determine the existence of the two periodic solutions corresponding to the MAWs via the equilibria of the averaged system by the averaged method for given integration constant.

Inserting (1.2) into (1.1), we obtain the following two coupled nonlinear ordinary differential equations

$$R'' + \tilde{\delta}R - \frac{c^2}{R^3} + \varepsilon\tilde{g}(x)R^3 + \varepsilon\tilde{V}(x)R = 0, \tag{1.3}$$

$$\Theta'' + 2\Theta' R'/R = 0 \Rightarrow \Theta'(x) = \frac{c}{R^2}, \tag{1.4}$$

where

$$\tilde{\delta} := 2\mu, \quad \varepsilon\tilde{g}(x) := -2g(x), \quad \varepsilon\tilde{V}(x) := -2V(x)$$

and the integration constant c, determined by the velocity and number density, plays the role of "angular momentum".

Inspecting Eq. (1.3) we know that for the simple case, i.e., $c = 0$, it is just the parametrically driven Duffing equation with the time variable replaced by the spatial coordinate, the MAWs in this system have been widely researched [10,15]. In the general case, $c \neq 0$, the system with regularities becomes more complicated and the MAWs may be kept [13].

For notational convenience, we drop the tildes from $\tilde{\delta}, \tilde{g}$ and $\tilde{V}$, and without loss of generality, we take the integration constant $c = 1$, so that (1.3) is written in the form of a forced second-order ODE as

$$R'' + \delta R - \frac{1}{R^3} + \varepsilon g(x)R^3 + \varepsilon V(x)R = 0. \tag{1.5}$$

Similarly, for convenience we move the coordinate $x = 0$ to $x = -c_0 := \delta^{1/4}$, then Eq. (1.5) is equivalent to the following planar Hamiltonian system

$$\begin{cases} R' = S \\ S' = -\delta(R - c_0) + \frac{1}{(R-c_0)^3} - \varepsilon g(x)(R - c_0)^3 - \varepsilon V(x)(R - c_0). \end{cases} \tag{1.6}$$

In this paper, we consider the case that both $V(x)$ and $g(x)$ are periodic functions (periodic potential and scattering length subjected to a spatially periodic modulation), and $\delta > 0$ corresponding to a positive chemical potential.

The rest of paper is organized as follows. In Section 2, at first we introduce the change of action-angle variables, then transform equation (1.6) to a standard form of averaging. The existence of period solutions will be determined by the equilibria of the averaged system. In Section 3 we construct a series of canonical transformations via generating functions to transform the original Hamiltonian into a simple form. A good approximation of order one for MAWs is obtained.

2. Construction of action and angle variables

We carry out the standard reduction for Eq. (1.6) to the action and angle variables [16]. In order to introduce action and angle variables, we consider the auxiliary autonomous system

$$R' = S, \quad S' = -\delta(R + c_0) + \frac{1}{(R + c_0)^3}. \tag{2.1}$$

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