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Coherence properties of cycling chaos

T.A. Levanova [a], G.V. Osipov [a], A. Pikovsky [b,*]

[a] Department of Control Theory, Nizhni Novgorod State University, Gagarin Av. 23, 606950 Nizhni Novgorod, Russia
[b] Institute for Physics and Astronomy, University of Potsdam, Karl-Liebknecht-Str. 24-25, 14476 Potsdam, Germany

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ABSTRACT

Cycling chaos is a heteroclinic connection between several chaotic attractors, at which switchings between the chaotic sets occur at growing time intervals. Here we characterize the coherence properties of these switchings, considering nearly periodic regimes that appear close to the cycling chaos due to imperfections or to instability. Using numerical simulations of coupled Lorenz, Roessler, and logistic map models, we show that the coherence is high in the case of imperfection (so that asymptotically the cycling chaos is very regular), while it is low close to instability of the cycling chaos.

1. Introduction

Heteroclinic cycles have attracted a lot of interest recently. This phenomenon was first introduced by Guckenheimer and Holmes [1,2]. Heteroclinic cycles are stable regimes of switches between meta-stable states, where these states are observed during longer and longer periods of time, so that the cycle period grows indefinitely. In the phase space of the system, a trajectory passes sequentially through vicinities of saddles, approaching the limiting closed orbit composed of heteroclinic pieces [3,4]. Physically, heteroclinic cycle appears as a sequential excitation of system's elements [5]. The cases of heteroclinic cycles based on saddle equilibria [6] or saddle cycles [7–9] and heteroclinic orbits connecting them are well studied.

An interesting variant of a heteroclinic cycle is cycling chaos, first described by Dellnitz et al. [10] and then studied in Refs. [11,12,4]. Here the saddle states, which a trajectory approaches in the course of evolution, are chaotic states. In the simplest setup one observes sequential periods of chaotic activity of the participating systems, interrupted by epochs of quiescence. While the internal oscillations are strongly irregular, the switching has a large degree of regularity.

In this paper we study coherence properties of the cycling chaos, i.e. we characterize its irregularity. The concept of coherence is well-defined for nearly periodic oscillators, for which the phase grows on average uniformly; coherence is determined by relative deviations from the uniform growth. Quantitatively, one defines the diffusion constant of the phase and compares it with the mean frequency (the same approach works for chaotic oscillators with well-defined phase). Cycling chaos is not a stationary process, as the "periods" grow in time. Therefore, to be able to apply the notion of coherence, we consider statistically stationary regimes that are close to the cycling chaos. We study two types of perturbations of the cycling chaos state that lead to stationary oscillations. The first type is imperfection in the equations that breaks invariance of the sets when only one subsystem constituting cycling chaos is excited and other ones vanish. Another imperfection is in fact a bifurcation at which the cycling chaos becomes unstable and a stable stationary regime in its vicinity appears. In both these situations, we characterize the irregularity of the switchings by the variation of their periods, which is equivalent to the phase diffusion constant, and study how it depends on the parameters. We are especially interested in the range of parameters where the mean period grows and the stationary cycle becomes closer and closer to the heteroclinic one.

* Corresponding author. Tel.: +49 331 977 1472.
E-mail address: pikovsky@uni-potsdam.de (A. Pikovsky).

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The paper is organized as follows. In Section 2 we introduce three models of cycling chaos. Two of them are based on three coupled continuous Lorenz and Roessler oscillators. The third model is a discrete and based on three coupled logistic maps. We also discuss the instability transition and the effect of imperfections. Then we study the statistical properties of these models in Section 3. In Section 4 we show that similar effects are observed in the cycling chaos of four Lorenz systems.

2. Models of cycling chaos

All models of cycling chaos that we use below are based on standard models of chaos, interaction of which is organized to ensure cycling. The first model is three coupled Lorenz systems

$$\begin{aligned}\dot{x}_i &= \sigma(y_i - x_i)\\ \dot{y}_i &= R_i x_i - y_i - x_i z_i + \delta\\ \dot{z}_i &= -b z_i + x_i y_i + \delta\end{aligned} \tag{1}$$

where $i = 1, 2, 3$ and parameters $\sigma = 10,\ b = 8/3$ are the standard ones. The coupling is via the dependence of parameters R_i on the states of the other oscillators:

$$R_i = R(1 - \alpha z_{i+1} - \beta z_{i-1}) \tag{2}$$

where $R = 28$ and periodicity in the index is assumed (so that $z_4 \equiv z_1$, $z_0 \equiv z_3$). Parameters α and β, as we show next, are responsible for the stability of chaotic cycling. The small parameter δ describes non-perfectness of the heteroclinic cycle.

Let us start with a perfect situation $\delta = 0$. In this case system (1) possesses three symmetric invariant manifolds on which one Lorenz oscillator is excited ($x_i, y_i, z_i \neq 0$) while other two are zero equilibrium ($x_{i\pm1} = y_{i\pm1} = z_{i\pm1} = 0$). The condition for the stable heteroclinic cycle, according to theory [12], is the following: these manifolds should be of saddle type, i.e. while the zero state in one subsystem is stable with negative transverse Lyapunov exponent λ_-, the zero state in another subsystem should be unstable with positive transverse Lyapunov exponent λ_+. Moreover, for the heteroclinic cycle to be attracting, the stability should be stronger then repulsion: $|\lambda_-| > |\lambda_+|$. These Lyapunov exponents are governed by the parameters $\alpha,\ \beta$. Indeed, linear stability of the zero states is governed by the system

$$\begin{aligned}\dot{x} &= \sigma(y - x)\\ \dot{y} &= R(1 - sZ(t))x - y\\ \dot{z} &= -bz\end{aligned} \tag{3}$$

where s is α or β depending on which neighbor of the excited system (from which the value of $Z(t)$ is taken) is considered. Calculation of the largest Lyapunov exponent of the linear system (3) gives results presented in Fig. 1. For $s = s_1 = 0.16$ and $s = s_2 \approx 0.0263$ the transverse Lyapunov exponents are equal in absolute value. Thus, below we set $\beta = s_1 = 0.16$, and then the heteroclinic cycle will be stable for $\alpha > s_2$ and unstable for $\alpha < \alpha_c = s_2$.

The dynamics of system (1) is illustrated in Fig. 2. The panel (a) shows the dynamics of variables z_i for $\delta = 0$ and $\alpha = 0.03 > \alpha_c$. For these parameters there exist three invariant chaotic sets, where one Lorenz system is active (e.g., $x_1, y_1, z_1 \neq 0$), while two other vanish (e.g., $x_i = y_i = z_i = 0$ for $i = 2, 3$). These sets are transversally saddles, forming an attracting heteroclinic cycle. One can see how a trajectory approaches this cycle, staying at each of chaotic sets for longer and longer times.

The panel (b) shows the regime for $\alpha = 0.03$ and $\delta = 10^{-20}$. With the imperfection $\delta \neq 0$ the non-active Lorenz systems do not vanish, but their variables have values $\approx \delta$. Correspondingly, the trajectory approaches a cycling orbit, period of which is bounded from above, this period tending to infinity as non-perfectness δ decreases.

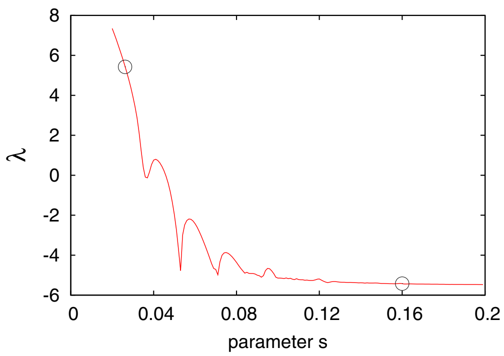

Fig. 1. Transverse Lyapunov exponents for the Lorenz model (1) in dependence on parameter s. Circles show two values of s for which the heteroclinic cycle is neutrally stable $|\lambda_-| = |\lambda_+| \approx 5.4252$.