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Border collision bifurcations and power spectral density of chaotic signals generated by one-dimensional discontinuous piecewise linear maps

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ABSTRACT

Recently, many papers have appeared which study the power spectral density (PSD) of signals issued from some specific maps. This interest in the PSD is due to the importance of frequency in the telecommunications and transmission security. With the large number of wireless systems, the availability of frequencies for transmission and reception is increasingly uncommon for wireless communications. Also, guided media have limitations related to the bandwidth of a signal. In this paper, we investigate some properties associated to the border-collision bifurcations in a one-dimensional piecewise-linear map with three slopes and two parameters. We derive analytical expressions for the autocorrelation sequence, power spectral density (PSD) of chaotic signals generated by our piecewise-linear map. We prove the existence of strong relation between different types of the power spectral density (low-pass, high-pass or band-stop) and the parameters. We also find a relation between the type of spectrum and the order of attractive cycles which are located after the border collision bifurcation between chaos and cycles.

1. Introduction

Chaotic signals are deterministic, non-periodic and random-like signals derived from nonlinear dynamical systems, they have sensitivity to initial conditions (SIC) [1]. The sensitivity to initial conditions means that the signals obtained with similar initial conditions can become very different when the time passes. These properties of chaotic signals can be exploited in several fields such as modulation, voice processing and telecommunications [2,3]. In recent years, several work appear interested in the spectral analysis of different chaotic maps due to insufficient frequencies available for communication and limited frequency bands in guided media [4,5].

In [6], Tsekeridou et al. have detailed the calculation of the autocorrelation and the spectrum of the Bernoulli map for use in watermarking. This Bernoulli watermarks can be generated with low-pass spectral properties to withstand low-pass attacks. In [5], Eisencraft and Kato have studied the skew tent map with a single parameter. They have found that when the bifurcation parameter varies the spectrum changes from low-pass spectrum to broadband then high-pass. Then, they have applied these different types of spectrum to modulation and demodulation. In [4], Eisencraft

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et al. have derived analytical expressions for the autocorrelation sequence (ACS), power spectral density (PSD) and essential bandwidth of chaotic signals generated by the skew tent map and they have deduced that the bandwidth of skew tent map is related with the Lyapunov exponents (the bandwidth is wider when the Lyapunov exponent increases). In [7,8], we have previously studied a discontinuous piecewise linear map and also discussed the possibility to obtain chaotic signals with different types of bandwidth, using numerical simulations. In [9], we have derived analytical expressions for the ACS, PSD and essential bandwidth of chaotic signals generated by one dimensional continuous piecewise linear maps with three slopes.

Our aim would be to generalize the results of [4,9] to a family of piecewise linear maps (PWLM) with different numbers of slopes or find other types of spectrum that can be managed by the parameters of the map. PWLM are of high interest because previous studies have shown that chaos exists for whole intervals of parameters and it is very easy to build iterated chaotic sequences with such maps. Another interest is that it provides different kinds of spectrum and gives more choice to generate spectrum for applications. Unfortunately, results are not trivial to obtain and analytical calculations have to be explicitly done for each kind of map. The complexity increases with the number of different slopes in the map.

In this paper, we propose to extend and complete previous work. Therefore, we define a class of maps dependent on two real parameters (β, Φ) for which we can calculate analytically the ACS and PSD for one case ($\beta = -\Phi$). The results are not so easy to obtain, indeed it is necessary to consider the intervals where the maps have different slopes. Then, we try to find a relation between the type of spectrum and the order of cycles which are involved in the border collision bifurcation giving rise to chaos. This paper is organized as follows. In Section 2 we introduce a one-dimensional piecewise linear map with two parameters and we study the border collision bifurcations (BCB). New techniques for obtaining the PSD of chaotic signals are discussed in Section 3. In Section 4 we simulate several autocorrelation sequence and power spectral density with different parameters; we end the paper by a conclusion.

2. One-dimensional piecewise linear map with two parameters

We are more particularly interested in one-dimensional maps, having the form:

$$x_{n+1} = T_{(\Phi,\beta)}(x_n), \tag{1}$$

with $n \in \mathbf{N}; x_n \in I$ interval of $\mathbf{R}; \Phi, \beta$ are two real parameters.

As in [7,8], we try to find a relation between the different possible forms of chaotic attractors, bifurcation diagram and the power spectral density (PSD). In this paper, we consider the following general map $T_{(\Phi,\beta)} : [-1,1] \to [-1,1]$,

$$T_{(\Phi,\beta)}(x) = \begin{cases} f_1(x) = \frac{x}{\Phi+1} + \frac{1}{\Phi+1}, & \text{if } x \in [\min(-1,\Phi),\Phi] \cap [-1,1], \\ f_2(x) = \frac{-2x}{-\beta+\Phi} + \frac{\Phi+\beta}{-\beta+\Phi}, & \text{if } x \in [\Phi, \max(\Phi,\beta)[\cap[-1,1], \\ f_3(x) = -\frac{x}{\beta-1} + \frac{1}{\beta-1}, & \text{if } x \in [\max(\Phi,\beta), \max(\beta,1)] \cap [-1,1], \end{cases} \tag{2}$$

which depends on real parameters Φ and β. When parameters are changed, we can obtain different functions with two or three slopes (cf. Fig. 1). The initial condition x_0 is chosen in the interval $I =]-1,1[$.

2.1. Bifurcation diagram

A fundamental problem of nonlinear dynamics is the study of bifurcations in the parameter space. A bifurcation corresponds to a qualitative change in the behavior of the system when one parameter (in our case Φ or β), crosses through a critical value Φ_c or β_c. A bifurcation may correspond to the appearance or disappearance of new singularities, the change in the stability of the system or the change in the shape of a chaotic attractor.

Fig. 1 shows the bifurcation diagram for the map (2) in the parameter plane (Φ, β). Specific bifurcations that appear in piecewise linear maps (2) are often border collision bifurcations. This type of bifurcations corresponds to a contact between an invariant set and the boundary of the region of differentiability [10]. The term border collision bifurcation appears for the first time in the work by Nusse and Yorke [11,12] and is now widely known for piecewise smooth maps [13,14]. Among the works for this type of bifurcation we can find Leonov who described and gave a recursive relation for finding the analytical expression of the sequence of bifurcations that occur in a one-dimensional piecewise linear map with a point of discontinuity [15,16]. Other studies on piecewise linear maps can be found in [17–19]. Other authors have described the bifurcations contact now called border collision, but using different names and notations [20,21].

To determine the fixed points and the n-cycles, we propose to write the map $T_{(\Phi,\beta)}$ using different notations $T_1, \ldots, T_5$, depending on the location of the parameters Φ and β (Fig. 1).

When $-1 < \Phi < \beta < 1$, we obtain the map T_1:

$$T_1(x) = \begin{cases} f_1(x), & \text{if } -1 \leqslant x \leqslant \Phi, \\ f_2(x), & \text{if } \Phi < x \leqslant \beta, \\ f_3(x), & \text{if } \beta < x \leqslant 1. \end{cases} \tag{3}$$