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Lag projective synchronization of a class of complex network constituted nodes with chaotic behavior

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ABSTRACT

In this paper, a method of the lag projective synchronization of a class of complex network constituted nodes with chaotic behavior is proposed. Discrete chaotic systems are taken as nodes to constitute a complex network and the topological structure of the network can be arbitrary. Considering that the lag effect between network node and chaos signal of target system, the control input of the network and the identification law of adjustment parameters are designed based on Lyapunov theorem. The synchronization criteria are easily verified.

1. Introduction

Watts and Strogatz established the famous small-world network model in 1998 and described its characteristics [1], leading that the investigation of complex network has attracted the comprehensive attention. After that, Barabási and Albert established the scale-free networks model [2], which drives further development of the research on complex network theory. With further research, the research interests have been expanded gradually from discovery of complex network model, investigation of its characteristics to prediction of network behavior and improvement of network performance. Especially, the synchronization performance of the complex network has shown unique application potential, and up to now, it plays a very important role in the many fields, such as nuclear magnetic resonance, laser transportation, information and communication, Internet, and so on. Therefore, the research on synchronization of the complex network has become a hotspot of network dynamics theory [3–7].

The groundbreaking work of the systematic investigation of complex network synchronization is marked by Pecora and Carroll, and they analyzed the synchronization stability of linear-coupled network and proposed the Master Stability Functions (MSF) criterion [8]. The MSF criterion of network is expanded and the synchronization problem of complex network with nondiagonized Lapalcae matrix is analyzed in Ref. [9]. The synchronization characteristics of a class of time-varying network are studied because the changes of connection structure between the network nodes with time often exist in practical applications [10]. The relationship between characteristic quantity of network structure and network synchronization performance are fully discussed [11], which leads that the effective way to improve network synchronization capability is obtained. In recent years, there are a large of literatures which focus on the chaos synchronization of complex network and improvement of synchronization performance, and the types of network synchronization are not only the complete synchronization [12–15], but also the phase synchronization [16,17] and generalized synchronization [18,19]. These fruitful

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works establish a solid foundation for the theoretical research and application of chaos synchronization of complex networks.

One of the most valuable issues in the investigation of the complex network synchronization is projective synchronization. So-called projective synchronization of network is the situation that each node in complex network can track the chaotic signals of the target system according to the magnitude of the scaling factor. At present, the working mechanism of a single chaotic system to track any given external input signal according to the magnitude of the scaling factor is relatively mature. However, the investigations about the projective synchronization among a number of interrelated chaotic systems and any given external input signal are still relatively rare. Particularly, the lag projective synchronization of a class of discrete network with uncertain adjustment parameters has not been reported. In this work, the discrete chaotic systems are taken as nodes to constitute a complex network, and the lag projective synchronization of the complex network is investigated. Considering that the lag effect between network nodes and chaos signal of target system, the control input of the network and the identification law of adjustment parameters are designed based on Lyapunov theorem. In simulation, the discrete Henon systems are taken as nodes to constitute the complex network, and the Ikeda system is taken as the target system. Lag projective synchronization between the network and the target system is proved and realized by simulation which shows that the synchronization technique is effective.

2. Mechanism of network lag projective synchronization

Consider a discrete chaotic system

$$x(n+1) = F(x(n)) = ax(n) + f(x(n)) \tag{1}$$

where n is discrete time series, $x(n) \in R^m$ are state variables of system, $F : R^m \to R^m$. a is the coefficient of linear term of the system.

The N-discrete chaotic systems (1) are taken as nodes to constitute a network, and node i satisfies the following state equation

$$x_i(n+1) = ax_i(n) + f(x_i(n)) + \varepsilon_i \sum_{j=1}^{N} c_{ij} x_j(n) + u_i(n) \quad (i = 1, 2, \ldots, N) \tag{2}$$

where ε_i is the coupling strength between the network nodes, c_{ij} is matrix element of the coupling matrix C and it represents the topological structure of the network. In this work, the topological structure of the network can be arbitrary. And if there is a connection between node i and j in the network, then $c_{ij} = c_{ji} = 1 (i \neq j)$; otherwise, $c_{ij} = c_{ji} = 0 (i \neq j)$. $u_i(n)$ is the control input of the network.

Assuming that the target system is

$$s(n+1) = bs(n) + g(s(n)) \tag{3}$$

where b is the coefficient of linear term of the target system.

Definition of the error between network and target system is

$$e_i(n) = x_i(n) - \sigma_i s(n - \tau) \qquad (i = 1, 2, \ldots, N) \tag{4}$$

where σ_i is scale factor of projective synchronization of network nodes, τ is lag value.

The evolution equation of the error can be further obtained

$$e_i(n+1) = x_i(n+1) - \sigma_i s(n+1-\tau) = ae_i(n) + f(x_i(n)) + \varepsilon_i \sum_{j=1}^{N} c_{ij} x_j(n) + u_i(n) + \sigma_i (a-b) s(n-\tau) - \sigma_i g(s(n-\tau)) \tag{5}$$

Designing the form of the control input of the network as

$$u_i(n) = -f(x_i(n)) - \varepsilon_i \sum_{j=1}^{N} c_{ij} x_j(n) - \sigma_i (a-b) s(n-\tau) + \sigma_i g(s(n-\tau)) - k_i e_i(n) \tag{6}$$

where k_i is uncertain adjustment parameter.

Considering Eq. (6), Eq. (5) can be simplified as

$$e_i(n+1) = (a - k_i) e_i(n) \tag{7}$$

There must be $e_i(n) \to 0$ if the following expression exists when $n \to \infty$

$$|e_i(n+1)| < |e_i(n)| \tag{8}$$

i.e.

$$|a - k_i| < 1 \tag{9}$$