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An equivalent linearization technique for nonlinear piezoelectric energy harvesters under Gaussian white noise

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ABSTRACT

An equivalent linearization technique is proposed to determine approximately the output voltage a nonlinear piezoelectric energy harvester excited by Gaussian white noise excitations. Equivalent linear system is derived from minimizing the mean-squared of the error. The linear equivalent coefficients are presented by the method of normal truncation. The exact solution of equivalent linear system is derived obtained. The effectiveness of the method is demonstrated by numerical simulations.

1. Introduction

Energy harvesting from ambient waste energy for the purpose of running low-powered electronics has emerged as a prominent research area and continues to grow at the rapid pace. As a promising approach, the piezoelectricity has been used to convert ambient vibration into useful electrical energy. There are several excellent and comprehensive survey papers and monographs, notably Sodano [1], Anton and Sodano [2], Priya [3], Tang et al. [4], Priya and Inman [5] and Erturk and Inman [6], reviewing the state of the art in different time phases of investigations related to piezoelectric energy harvesting.

Concentrating on the resonance under a harmonic excitation, most works on energy harvesting took the deterministic approach. Especially, nonlinearity was introduced to increase the operating frequency range of energy harvesters. Stanton et al. [7] applied the method of harmonic balance to characterize quantitatively the beam and electrical network oscillation amplitudes, and validated the response amplitudes of numerical simulations' predicated by experiments. They investigated the response of harvesting energy as a nonlinear oscillator, and demonstrated that the bistability may be used to improve energy harvesting [8]. Erturk et al. [9], Erturk and Inman [10] constructed a piezomagnetoelastic energy harvester, investigated numerically and experimentally the response to harmonic excitations. Triplett and Quinn [11] considered a nonlinear piezoelectric relationship on the performance of a vibration-based energy harvester, used Poincare–Lindstedt perturbation method to analysis the response of the harvesting system.

Despite the successfulness of the deterministic approach, randomness, inherent in most real-world circumstances, may significantly change the behavior of vibration-based energy harvesters. There are some researches via stochastic approaches. Establishing the closed-form expressions for output power, proof mass, displacement, and optimal load for linear energy harvesters driven by broadband random vibrations, Halvorsen [12] demonstrated that the output power has a different optimum for broadband excitations from that for sinusoidal excitations. McInnes et al. [13] employed stochastic resonance to

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enhance vibration energy harvesting and revealed numerically the significant enhancement without any periodic forcing. Cottone et al. [14] found numerically and experimentally that the nonlinear oscillators can outperform the linear ones under stochastic excitation. Based on a single-degree-of-freedom model, Adhikari et al. [15] analyzed the mean power of a linear piezoelectric energy harvester under stationary Gaussian white noise. Gammaitoni et al. [16] revealed nonlinear oscillators can outperform the linear ones under noise excitation in monostable configurations. However, Daqaq [17] demonstrated monostable Duffing oscillator does not provide any enhancement over the typical linear oscillators under white Gaussian and colored excitations. Litak et al. [18] calculated the response of a nonlinear piezomagnetoelastic energy harvester under stationary Gaussian white noise. Daqaq [19] derived an approximate expression for the mean power under exponentially correlated noise and demonstrated the existence of an optimal potential shape maximizing the output power. Analyzing a piezomagnetoelastic energy harvester of strongly nonlinear under random excitations, Ali et al. [20] established a closed-form approximate expression of the harvested power and validated against numerical Monte Carlo simulation results. Zhu and Zu [21] presented a new magnetoelectric energy harvester from nonlinear vibrations by magnetic levitation, derived the government equations of the model and obtained the mechanical and electrical responses in time-domain. Green et al. [22] reported Duffing-type nonlinearities can reduce the size of energy harvesting devices without affecting their power output, verified the result using the technique of equivalent linearization.

Among various approaches nonlinear random vibration, the equivalent linearization is a simplest in principle and easiest in practice. The basic idea of equivalent linearization techniques is to replace a given nonlinear stochastic system, for which the exact stationary solution is not obtainable analytically, with a linear stochastic system, whose behavior is closest to the given one in some statistical sense and whose exact stationary solution is obtainable. It is becoming a convenient approximate approach to predict the stationary response of nonlinear stochastic systems. Historically, Caughey [23] proposed the equivalent linearization technique for a nonlinear oscillator subjected to stationary Gaussian random excitation. Iwan and Yang [24] extended the equivalent linearization technique to nonlinear multi-degree-of-freedom systems. Spanos [25] developed stochastic linearization for symmetric or asymmetric nonlinear systems, outlined the solution procedure for determining nonstationary or stationary system response statistics.

It should be remarked that there are different types of electrical circuit equations used in piezoelectric energy harvesting. duToil et al. [26] first proposed a coupled electromechanical equation for lumped-parameter piezoelectric energy harvesters. As the motion equation of the harvester and its electromechanical equation cannot be directly decoupled, the system is with 1.5 degrees-of-freedom. This type of the electrical circuit equation has been widely used [7–11,12–16,18,20]. Daqaq [17,19] and Green et al. [22] introduced an uncoupled electrical circuit equation, the electromechanical equation can be converted into first-order differential equation. Therefore, the system is with single-degree-of-freedom. Triplett and Quinn [11] considered a nonlinear piezoelectric coupling relationship on the performance of a vibration-based energy harvester. Adhikari et al. [15] reported an electrical circuit equation with an inductor, where the electrical equation is second-order differential equation. Thus, the system is with 2 degrees-of-freedom. This paper investigated the lumped-parameter model of a piezoelectric energy harvester which is essentially a 1.5 degree-of-freedom system. So far, to the authors' best knowledge, there is no equivalent linearization analysis on such a system. To address the lacks of research in the aspect, the present work develops the equivalent linearization technique to determine the response of nonlinear piezoelectric energy harvesters under Gaussian white noise excitation.

The paper is organized as follows. Section 2 presents the governing equation of piezoelectric coupling systems under Gaussian white noise excitation. Section 3 derives the equivalent linear system from the criteria of least mean-squared error, and gives the exact solutions of equivalent linear system. Section 4 analyzes the validity of the method via numerical simulations. Section 5 ends the paper with concluding remarks.

2. The governing equation

The lumped-parameter model of a piezoelectric energy harvester with cubic nonlinearity in the displacement term under Gaussian white noise excitation can be given as

$$\ddot{x} + 2\zeta\dot{x} + \omega_0^2 x + \varepsilon\alpha x^3 - \chi\upsilon = \xi(t), \tag{1}$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0, \tag{2}$$

where x is the displacement response, υ is the voltage response across the external electrical load, ω_0 is the undamped fundamental natural frequency, χ is the piezoelectric coupling term in the mechanical equation, λ is the reciprocal of the time constant of the resistive–capacitive circuit, κ is the piezoelectric coupling term in the electrical equation. Furthermore, ε is a small bookkeeping parameter and ζ is a mechanical damping term, $\xi(t)$ is stationary Gaussian white noise process with zero mean and autocorrelation function

$$\langle \xi(t)\xi(t+\tau)\rangle = 2\pi S_0\delta(\tau), \tag{3}$$

where $\langle\rangle$ denotes the expected value, S_0 is the spectral density of the excitation, and δ is the Dirac function.