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Frequency–energy plots of steady-state solutions for forced and damped systems, and vibration isolation by nonlinear mode localization

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ABSTRACT

We study the structure of the periodic steady-state solutions of forced and damped strongly nonlinear coupled oscillators in the frequency–energy domain by constructing forced and damped frequency – energy plots (FEPs). Specifically, we analyze the steady periodic responses of a two degree-of-freedom system consisting of a grounded forced linear damped oscillator weakly coupled to a strongly nonlinear attachment under condition of 1:1 resonance. By performing complexification/averaging analysis we develop analytical approximations for strongly nonlinear steady-state responses. As an application, we examine vibration isolation of a harmonically forced linear oscillator by transferring and confining the steady-state vibration energy to the weakly coupled strongly nonlinear attachment, thereby drastically reducing its steady-state response. By comparing the nonlinear steady-state response of the linear oscillator to its corresponding frequency response function in the absence of a nonlinear attachment we demonstrate the efficacy of drastic vibration reduction through steady-state nonlinear targeted energy transfer. Hence, our study has practical implications for the effective passive vibration isolation of forced oscillators.

1. Introduction

In recent studies the free and forced dynamics of strongly nonlinear oscillators have been considered; i.e., of dynamical systems with essential stiffness nonlinearities possessing negligible or very small linear components [6–8,15,17]. The high degeneracy that this class of strongly nonlinear systems possesses gives rise to interesting nonlinear dynamical phenomena, such as cascades of transient resonance captures [2,16], broadband vibration energy transfers between subcomponents (targeted energy transfers) [17], and nonlinear localization phenomena [4]. Tools for analyzing the strongly nonlinear dynamics of these systems have been developed, such as wavelet spectra superpositions on frequency–energy plots – FEPs of Hamiltonian dynamics and complexification/averaging analysis [17]. As shown in previous works two-dimensional FEPs provide a synoptic global description of the frequency and energy dependencies of periodic orbits of Hamiltonian n-degree of freedom

Acronyms: DOF, Degree-of-freedom; FEP, Frequency-energy plot; NNM, Nonlinear normal mode; LO, Linear Oscillator; NES, Nonlinear Energy Sink; CX-A, Complexification-Averaging Method.

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(DOF) dynamical systems, and can be used to interpret complex dynamical transitions of weakly damped systems possessing even strong, non-smooth nonlinearities [9]. This is achieved by superimposing wavelet spectra of the damped responses onto the Hamiltonian FEPs, thus identifying the underlying Hamiltonian dynamics that influence the damped transitions [17].

The principal aim of the present work is to study the perturbations of the Hamiltonian FEPs in the presence of weak damping and small-amplitude harmonic excitation. This topic has not been addressed in the literature and, hence, the results reported here are new. Considering a specific two-DOF oscillator with strong nonlinearity, our analysis is carried out under conditions of 1:1 resonance; i.e., when the two degrees of freedom oscillate in synchronicity with identical dominant frequencies. Moreover, the applied harmonic excitation is assumed to also possess a frequency close to the frequency of oscillation of the system, so an additional condition of fundamental resonance is assumed. Our analysis is based on complexification-averaging (CX-A) of the equations of motion, and the results are applied to the problem of optimizing vibration isolation of a harmonically forced linear system by means of transferring steady-state energy to a weakly coupled strongly nonlinear attachment (termed the nonlinear energy sink – NES).

Whereas interesting recent contributions to the problem of optimal vibration isolation through nonlinear targeted energy transfer have appeared [5,13,14], our approach to the optimization is different since it is based on the forced and damped FEP of the system. We note that a two-DOF system similar to the one considered in this work was first considered in [3], where the feasibility of steady-state energy transfer from the directly excited linear oscillator to the nonlinear attachment was addressed without performing, however, any optimization related to vibration isolation. In fact, as pointed out in [10] where this system was re-examined, a Hopf bifurcation in the steady-state dynamics of the weakly coupled system leading to weakly modulated responses was missed in [3]. However, what we show in the present work is that optimization of the steady-state dynamics of the weakly coupled and weakly damped two-DOF system of [3] can lead to effective vibration isolation by means of steady-state nonlinear targeted energy transfer and localization.

2. Forced and damped frequency – energy plots (FEPs)

We consider in this paper a two-DOF system, consisting of a weakly damped linear oscillator – LO weakly coupled with an essentially nonlinear, weakly damped attachment (which will act as a nonlinear energy sink – NES) through a weak linear stiffness of constant εk_c, where $|\varepsilon|<<1$ is a small quantity that will be designated as the *perturbation parameter* of our study. The stiffness nonlinearity is characterized as *essential* since its characteristic is purely cubic (with coefficient k_{nl}) and lacking a linear part; hence, the nonlinearity is completely *nonlinearizable*. The LO is excited by a small-amplitude harmonic force with amplitude equaling εP and frequency ω. The configuration of the system is depicted in Fig. 1. We are mainly interested in the amplitude-frequency dependence of the steady-state responses of the LO and the NES.

We will study the steady-state dynamics of this system both analytically and numerically. First, under the assumption of fundamental resonance and 1:1 resonance between the LO and the NES, we apply the CX-A method and slow/fast partition of the steady-state dynamics and construct damped and forced perturbations of the FEP of the underlying Hamiltonian system (corresponding to no damping or forcing). We note that our analytical approach applies even in this strongly nonlinear case, when traditional asymptotic methods of nonlinear dynamics based on the assumption of weak nonlinearity and linear generating solutions are not valid (since the current problem is non-linearizable). Then, we will verify the analytical results by direct numerical simulation of the equations of motion, to highlight some interesting phenomena of practical interest.

The equations of motion of the system of Fig. 1 are expressed as

$$\begin{aligned} &\ddot{x}(t) + \varepsilon\lambda_1\dot{x}(t) + \omega_0^2 x(t) + \varepsilon\alpha_1[x(t) - v(t)] = \varepsilon P\sin\omega t \\ &\ddot{v}(t) + \varepsilon\lambda_2\dot{v}(t) + C_s v^3(t) - \varepsilon\alpha_1[x(t) - v(t)] = 0 \end{aligned} \tag{1}$$

where the normalized coefficients are given by $\lambda_1 = c_1/m_1$, $\lambda_2 = c_1/m_2$, $\alpha_1 = k_c/m_1$, $\alpha_2 = k_c/m_2$, $C = k_{nl}/m_2$ and $\omega_0^2 = k_1/m_1$. For simplicity, we make the assumption of equal masses $m_1 = m_2$, so that $\alpha_1 = \alpha_2$, since this assumption will help us reduce the complexity of the resulting analytical derivations. Furthermore, again without loss of generality we set the natural frequency of the LO equal to unity, $\omega_0^2 = 1$ (this can always be achieved by appropriate rescaling of the time variable in (1)).

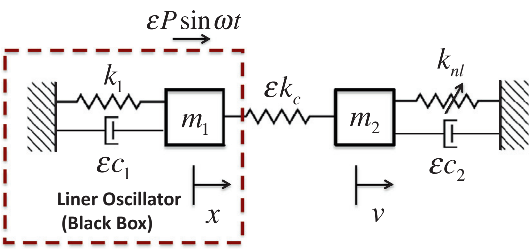

Fig. 1. Two-DOF weakly coupled and weakly damped system under weak harmonic forcing.