Commun Nonlinear Sci Numer Simulat 18 (2013) 2769–2774

Contents lists available at SciVerse ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Effect of spatial distribution on the synchronization in rings of coupled oscillators

Hongjing Ma [a], Weiqing Liu [b], Ye Wu [a,c], Yixian Yang [d], Jinghua Xiao [a,c,*]

[a] School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, China
[b] School of Science, Jiangxi University of Science and Technology, Ganzhou 341000, China
[c] State Key Lab of Information Photonics and Optical Communications, Beijing University of Posts and Telecommunications, Beijing 100876, China
[d] Information Security Center, Beijing University of Posts and Telecommunications, Beijing 100876, China

ARTICLE INFO

Article history:
Received 29 October 2012
Accepted 18 February 2013
Available online 14 March 2013

Keywords:
Log-normal distribution
Spatial distribution
Synchronizability

ABSTRACT

In this paper, the effects of spatial distribution of coupling on the synchronizability are explored in a ring of diffusively coupled oscillators. We find that the inhomogeneity and spatial arrangements of coupling strength have great impacts on the synchronizability. When the inhomogeneous coupling constants are spatially rearranged, the eigenvalues λ_2 (the second largest eigenvalue of the coupling matrixes) for all possible spatial arrangements, which may describe the synchronizability of coupled oscillators, obey a log-normal distribution. The spatial arrangement of period 1 achieves the best synchronizability while that of period 2 has the worst one. In addition, the regimes of the effects of spatial distribution on synchronizability are analyzed by a ring of coupled Rossler systems. The spatial rearrangement of coupling has meaningful applications in the manipulation of self- organization for coupled systems.

1. Introduction

Synchronization as one of collective behaviors for a population of dynamically coupled oscillators has been widely explored, mainly due to a better understanding the dynamics of many natural systems in different contexts such as biology, ecology, climatology, even in sociology[1,2]. Most of the studies focus on the phenomenon of complete synchronization [3–5] in the networks of identical nodes as well as the coherence of nonidentical phase oscillators [6]. The structure of the network plays a crucial role in determining the synchronization of coupled elements. The synchronizability of complex networks (such as the small-world (SW) and scale-free (SF) networks) is generally larger than that of the regular one due to their small distances [7,8]. Moreover, the SF network is found more difficult to be synchronized than the SW one though it has a smaller distance. However, by introducing the asymmetric coupling, the synchronizability of both the regular network [9] and the SF one can be significantly improved. The synchronizability of the SF network can even be much higher than that of the SW one. In a realistic network, the interactions between nodes are naturally weighted, e.g. the synchronization of epidemic outbreaks in the networks of cities[10]. The weights are found to have great influences on the synchronizability of the degree heterogeneous networks [11–13], where suitably weighted connections can significantly enhance the synchronizability without changing the heterogeneity in the topology. When the weights are properly correlated with the dynamics of nodes in the SF network, the synchronizability [14] can be further improved and becomes self-organized.

The model of the nearest coupled oscillators is widely used to describe the dynamics of pattern formation, amplitude death [15–17] and synchronization [9,18,19]. Since parameter mismatches is omnipresent in the real world, and it has a

* Corresponding author at: School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, China. Tel.: +86 01062282003.
E-mail address: jhxiao@bupt.edu.cn (J. Xiao).

1007-5704/$ - see front matter
http://dx.doi.org/10.1016/j.cnsns.2013.02.014

significant impact on the dynamics of the amplitude death and the synchronization. Moreover, the efficiency of synchronization can be enhanced by introducing an adaptive coupling [20] and by presenting proper spatial arrangements of frequency in a ring of coupled phase oscillators[21], where the synchronizability increases monotonously with a physical quantity roughness spatial frequency configuration. Hence, it is natural to raise the following questions. Does the spatial distribution of weights influence the synchronizability in a ring of coupled oscillators? What kind of spatial distribution of weights is beneficial for the synchronizability? To answer these questions, synchronization in a ring of coupled oscillators with different distributions of coupling constant is explored with a master-stability function method in this paper. We find an interesting result that λ_2 (the second largest eigenvalue of coupling matrixes), which can describe the synchronizability of coupled system for all possible spatial distributions of coupling constants, obeys a log-normal distribution. A spatial distribution of the coupling constant with a period 1 (2) is the easiest (hardest) way to achieve the synchronization.

The remainder of this paper is organized as follows. In section II, we give a model for synchronization. We analyze the effects of coupling mismatches on the synchronizability in section III. Section IV is devoted to explore the impacts of spatial distribution of coupling on the synchronizability. In section V, the regimes determining the impact of spatial arrangements on synchronizability are illustrated by using the coupled Rossler oscillators. Finally, section VI comes to the discussions and the conclusions.

2. Models

Let us consider a ring of bidirectionally coupled identical oscillators with weighted coupling

$$\dot{X}_i(t) = f(X_i) + \epsilon_i\Gamma(X_{i-1} - X_i) + \epsilon_{i+1}\Gamma(X_{i+1} - X_i), \quad i = 1, \ldots, N, \tag{1}$$

where $X_i \in \mathbf{R}^n$ $(i = 1, 2, \ldots, N)$, $f : \mathbf{R}^n \to \mathbf{R}^n$ is nonlinear and capable of exhibiting rich dynamics, e.g. chaos, ϵ_i is a scalar coupling constant, and Γ is an $N \times N$ inner linking matrix. The boundary condition is arbitrarily set as a periodical boundary with $X_{N+1} = X_1$, $X_0 = X_N$. The synchronization can be determined by the eigenvalue analysis [18,19]. Suppose the synchronous chaos falls onto the synchronous manifold $X_1(t) = X_2(t) = \ldots = X_N(t) = s(t)$, which satisfies $\dot{s}(t) = f(s(t))$. Its stability can be analyzed by linearizing Eq. 1 at the synchronous chaos $s(t)$, then we get

$$\dot{\eta} = [Df(s(t))I + B\Gamma]\eta, \tag{2}$$

where $Df(s(t))$ is the Jacobian of f on $s(t)$, $\eta = (\eta_1, \eta_2, \ldots, \eta_N)'$ is a small perturbation on $s(t)$, I is an $N \times N$ unit matrix, B is an $N \times N$ coupling matrix.

$$B = \begin{pmatrix} -(\epsilon_1+\epsilon_2) & \epsilon_2 & \cdots & \cdots & \epsilon_1 \\ \epsilon_2 & -(\epsilon_2+\epsilon_3) & \epsilon_3 & \cdots & \cdots \\ \cdots & \epsilon_3 & -(\epsilon_3+\epsilon_4) & \epsilon_4 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \epsilon_1 & \cdots & \cdots & \epsilon_N & -(\epsilon_N+\epsilon_1) \end{pmatrix}.$$

Expanding η over the eigenfunctions of B, we have

$$\dot{\delta_k} = [Df(s(t))I + \alpha\Gamma]\delta_k. \tag{3}$$

If we take α as a parameter, the stable synchronous area of parameter α can be determined by a negative maximum Lyapunov exponent of Eq. 3 for any given f and Γ. The stable synchronization area can be presented as $[\alpha_2, \alpha_1]$. If the eigenvalues of B are arranged as $\lambda_N < \ldots < \lambda_2 < \lambda_1 = 0$, then the stable synchronization is built as $\frac{\lambda_N}{\lambda_2} < \frac{\alpha_2}{\alpha_1}$, and the synchronizability is governed by the ratio $\frac{\lambda_N}{\lambda_2}$. Especially, it is simply determined by the value of λ_2 when $\alpha_2 = \infty$ for proper f and Γ. For simplicity, we will consider the synchronizability of systems in the situation of $\alpha_2 = \infty$ in the following section, which shows the larger $-\lambda_2$ is, the better synchronizability is.

3. Effects of mismatches of coupling constant

Firstly consider a ring of coupled oscillators with a linear trend of coupling distribution

$$\epsilon_i = \epsilon_0 + \left(i - \frac{N+1}{2}\right)\delta\epsilon, \tag{4}$$

where ϵ_i is the coupling constant between oscillators $i-1$ and i for all $i = 1, 2, \ldots, N$ (ϵ_N is the coupling constant between nodes N and 1 since the system has a periodical boundary condition). ϵ_0 is arbitrarily set as a large value to guarantee that all ϵ_i are positive (e.g. $\epsilon_0 = 1000$). $\delta\epsilon = \epsilon_{i+1} - \epsilon_i$ is a coupling mismatch between neighbor couplings. When $\delta\epsilon = 0$, the system is an un-weighted ($r = 0$) coupled oscillators as in Ref. [19] whose synchronizability can be determined by the $\lambda_2^{(0)}$ (its value is related to the system size N). To investigate the effects of $\delta\epsilon$ on synchronizability, $\lambda_2' = |\lambda_2| - |\lambda_2^{(0)}|$ versus $\delta\epsilon$ for different system size N are calculated. From the Fig. 1, one may find that λ_2' is exponentially decreasing with the increment of coupling constant as in Eq. 5,