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Homogeneous feedback design of differential inclusions based on control Lyapunov functions

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ABSTRACT

This paper is concerned with the stabilization of differential inclusions. By using control Lyapunov functions, a design method of homogeneous controllers for differential equation systems is first addressed. Then, the design method is developed to two classes of differential inclusions without uncertainties: homogeneous differential inclusions and nonhomogeneous ones. By means of homogeneous domination theory, a homogeneous controller for differential inclusions with uncertainties is constructed. Comparing to the existing results in the literature, an improved formula of homogeneous controllers is proposed, and the difficulty of the controller design for uncertain differential inclusions is reduced. Finally, two simulation examples are given to verify the preset design.

1. Introduction

Over past decades, differential inclusions (DIs) have attracted much attention. In practice, DIs have been found many applications such as mechanical systems [1], electrical systems [2], hybrid systems [3], to list a few. Many interesting results have been obtained [4–8]. It should be noted that the research of DIs is mostly about linear differential inclusions (LDIs) [9,10], which have much simpler form and research method than nonlinear differential inclusions (NDIs). There are, however, many practical systems that cannot be described by LDIs [11,12], which implies the study on NDIs is very necessary. This paper focuses on the homogeneous feedback design of NDIs, which refers to homogeneous NDIs and nonhomogeneous NDIs. It is well known that, how to deal with nonlinear terms is an important but challenging work when modeling practical systems. Homogeneous approximation is regarded as one of effective approaches for solving that problem. Homogeneous differential equation systems and homogeneous differential inclusions (HDIs) naturally appear as two classes of approximate systems. Homogeneous control analysis and design have been widely studied [13–16]. Compared to homogeneous design of differential equation systems [17–20], there are few results on DIs. A construction method of a Lyapunov function associated was presented and a new definition of HDIs was also proposed in [21]. A homogeneous Lyapunov function for discrete-time HDIs was constructed in [22,23]. Note that HDIs considered in [21–23] are in the autonomous form and do not contain uncertainties. These give rise to some questions: Can the feedback design of non-autonomous HDIs be dealt with? Can the stabilization of NDIs with uncertainties be solved? If answers are positive, how to do? This is not a trivial work. First, the existing results on differential equation systems can not be directly applied to DIs. Second, the problems of DIs are harder than these of differential equation systems because DIs possess more complex system structure, which leads to the corresponding results of differential equation systems are not available to DIs. These inspire us carry out the work. This paper considers the homogeneous controller design of DIs by means of the control Lyapunov function (CLF) methodology and homogeneous domination theory. It is worth mentioning that CLF is a powerful tool for studying nonlinear problems because

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it can give a universal formula for constructing stabilizing controller. We first give a formula of homogeneous controller for nonlinear differential equation systems. The formula is then developed to HDIs without uncertainties and DIs with uncertainties. The rest of the paper is organized as follows: In Section 2, some fundamental preliminaries are introduced; Section 3 considers the stabilization of HDIs without uncertainties; In Section 4, the stabilization of DIs with uncertainties are solved based on the aforementioned conclusions; Section 5 gives simulation examples to illustrate the correctness of the results.

2. Preliminaries

This section is divided into two subsections. The first subsection introduces homogeneous domination theory and the second one gives the concepts of HDI and CLF.

2.1. Homogeneous domination theory

The following preliminaries about homogeneity can be found in [13,18,20].

Definition 1. For a given vector $x = (x_1, \ldots, x_n)^T \in \Re^n$, dilation Δ_λ^r is defined as

$$\Delta_\lambda^r(x) = (\lambda^{r_1}x_1, \ldots, \lambda^{r_n}x_n), \lambda \in \Re^+,$$

where $r_1, \ldots, r_n > 0$ are dilation coefficients and $\Re^+$ represents the set of positive real numbers.

Definition 2. A function $V(x) : \Re^n \to \Re$ is said to be homogeneous of degree τ if there is a real number $\tau \in \Re$ such that $V(\Delta_\lambda^r(x)) = \lambda^\tau V(x)$.

Definition 3. A vector field $f(x) = (f_1(x), \ldots, f_n(x))^T : \Re^n \to \Re^n$ is said to be homogeneous of degree k if there is a real number $k \in \Re$ such that

$$f_i(\Delta_\lambda^r(x)) = \lambda^{k+r_i} f_i(x),$$

for $i = 1, 2, \ldots, n$, where $\Delta_\lambda^r(x)$ is defined in Definition 1.

Definition 4. A homogeneous ϱ-norm is defined as

$$\|x\|_{\Delta,\varrho} = \left(\sum_{i=1}^{n} |x_i|^{\frac{\varrho}{r_i}}\right)^{\frac{1}{\varrho}}, \forall x \in \Re^n, \varrho \geqslant 1.$$

For simplicity, we denote $\|x\|_{\Delta,\varrho}$ by $\|x\|_\Delta$ throughout the paper.

Lemma 1. *If $V(x) : \Re^n \to \Re$ is a homogeneous function of degree τ, then*

(i) *There exists a constant c_1 such that $V(x) \leqslant c_1\|x\|_\Delta^\tau$. Moreover, if V is positive definite, $c_2\|x\|_\Delta^\tau \leqslant V(x)$, where c_2 is a positive constant.*
(ii) *$\frac{\partial V}{\partial x_i}$ is homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.*

Lemma 2. *Suppose $f_1(x)$ and $f_2(x)$ are homogeneous functions of degree k_1 and k_2 respectively with the same dilation Δ_λ^r. Then, $f_1(x)f_2(x)$ is also homogeneous function of degree $k_1 + k_2$ with the same dilation.*

2.2. HDI and CLF

In this subsection, we introduce the concepts of HDI and CLF.

Definition 6. A DI

$$\dot{x} \in F(x) + G(x)u(x) \tag{1}$$

is said to be an HDI of degree k_0 with respect to dilation Δ_λ^r if it satisfies

$$F_i(\Delta_\lambda^r(x)) = \lambda^{k_0+r_i} F_i(x), \quad i = 1, 2, \ldots, n,$$

$$G_j^i(\Delta_\lambda^r(x)) = \lambda^{k_1+r_i} G_j^i(x), \quad j = 1, 2, \ldots, m; \ \ i = 1, 2, \ldots, n,$$