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Application of three controls optimally in a vector-borne disease – a mathematical study

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ABSTRACT

We have proposed and analyzed a vector-borne disease model with three types of controls for the eradication of the disease. Four different classes for the human population namely susceptible, infected, recovered and vaccinated and two different classes for the vector populations namely susceptible and infected are considered. In the first part of our analysis the disease dynamics are described for fixed controls and some inferences have been drawn regarding the spread of the disease. Next the optimal control problem is formulated and solved considering control parameters as time dependent. Different possible combination of controls are used and their effectiveness are compared by numerical simulation.

1. Introduction

Mathematical modeling is an important tool to understand the dynamics of disease transmission and in decision making process regarding intervention programs for disease control. There is a long history of using mathematical models to describe infectious diseases and their controls (see [2,12,19–22], and references therein). Among all kind of diseases, vector borne diseases are one of the major growing issues in today's society. Diseases like malaria, dengue, filaria, yellow fever are examples of some of the vector borne diseases and these are spread out mainly by infected mosquitoes. So to remove these diseases from the society, it is not only the infected persons are to be recovered but also the infected vectors are to be killed or removed.

Malaria is one of the deadly vector borne diseases which spread among the humans by the infected mosquitoes. Though it is a global issue, but it is a major concern in non developed and developing countries rather than developed countries. According to the World Health Organizations (WHO) report (2009), there were 247 million cases of malaria in 2006, causing nearly one million deaths, mostly among African children. Beyond death, malaria impedes development in so many ways, it affects fertility, population growth, saving and investment, worker productivity, premature mortality and medical costs [17]. Malaria also affects fetal development during early stage of pregnancy in women due to loss of immunity. There are some good articles on the vector borne diseases like malaria, dengue, yellow fever etc. Nakul et al. [4] applied a mathematical model to determine some important parameters about the spread of malaria disease. Chiyaka et al. [1] considered treatment and spread of drug resistance to encounter malaria. Makinde and Okosun [5] derived and analyzed rigorously a mathematical model that describes the dynamics of malaria infection with the recruitment of infected immigrants, treatment of infectives and spray of insecticides against mosquitoes in the population. They also achieved the optimal strategies for disease control.

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Kawaguchi et al. (2004) examined the combined use of insecticide spray and zooprophylaxis as a strategy for malaria control. Jia (2008) formulated and examined a compartmental model for malaria transmission that includes incubation periods for both infected humans and mosquitoes. Mathematical models on some other vector borne diseases like dengue and yellow fever can be found in [15] and references therein. Lashari and Zaman [6] described the global dynamics of a vector borne diseases occurring due to the horizontal transmission.

In spite of being many deadly effects, malaria is preventable and curable when treatment and prevention measures are taken properly. Use of proper food and medicine, hospitalization etc. to the infectious individuals, are the mean of treatment control. Taking of suitable protection (e.g. vaccination), so that the parasites of these diseases could not enter in human body, is another way to keep the disease away. Since there is no proper medicine or drops to use as vaccine for these types of diseases, here the word 'vaccination' is used in a quite general sense. Here the use of bednets, mosquito coils and liquid, cream etc. so that the vector can not come to the contact of the susceptible human, can be considered as vaccination. Also these diseases will no more vulnerable to us if we can control the growth of the infected vectors. Application of insecticide to the vectors are the most effective way to control the increment of all vectors. Here by the word 'insecticide' we not only mean that the chemicals to be used to kill the vectors but also to create a proper environment that will be vulnerable to make the lifestyle of vector hard. Thus all the three controls, mentioned above can be used to control any vector borne disease. Further the most interesting observation of these types of diseases is that a susceptible person is not infected with the contact of an infected person, but if the susceptible vectors come into contact with the infected human then it becomes infected and spread the parasites of the disease to other susceptible human. Thus the number of infected persons totally depend on the contact of the susceptible human with the infected vectors and the quantity of the infected vectors depend on the contact between the susceptible vector and infected human. Also as we observe the exponential growth of mosquitoes or any other types of insects, unless they are controlled to be borne, the use of malthusian growth rate for the vector to be more logical.

Although there are some existing literatures on vector borne diseases including optimal control theory (see [3,7,9,18] and references therein), but to the authors knowledge no attempt is made to describe the phenomena of a vector borne disease considering all of the above described phenomena. So keeping all these issues in our mind, for the modeling of a vector-borne epidemic system we divide the total human population into four different classes namely susceptible (S), infected (I), recovered (R) and vaccinated (V), and total vector populations into two different classes namely the susceptible vector (L) and the infected vector (V). Let at any time t, a be the total recruitment of the human and among them u_1 $(0 \leqslant u_1 \leqslant 1)$ portion population are vaccinated. We assume that d is the natural death rate for the all classes of human population and γ be the additional death rate of the infected population due to the disease. Let m be the natural recovery rate from the infected population and among the natural recovers τ_1 portion go to the recovery class and the rest part of the human again becomes susceptible for that disease. u_2 $(0 \leqslant u_2 \leqslant 1)$ is the treatment control applied to the infected human population and due to this treatment, bu_2I portion of infected persons recover from the disease and among them τ_2 portion are taken in recovered class as they are not susceptible for that disease where as the rest portion are again become susceptible for the disease. Moreover we assume that among the recovered, the portion αR and among the vaccinated, the portion βV are the susceptible for the disease again. Let η and σ are respectively the parameters of transmission rate from susceptible human to infected human and susceptible vector to the infected vector. Also let r be the malthusian growth rate for the susceptible vector and δ be the natural death rate for the infected vector. We apply the control u_3 $(0 \leqslant u_3 \leqslant 1)$, to the vector, as insecticide and τ_3u_3L portion of susceptible vector and τ_4u_3M portion of infected vector are the victim of this control.

By means of the above assumptions we formulate our vector-borne epidemic model in the following way:

$$
\begin{aligned}
\frac{dS}{dt} &= (1-u_1)a - dS - \eta SM + \alpha R + m(1-\tau_1)I + bu_2(1-\tau_2)I + \beta V,\\
\frac{dI}{dt} &= \eta SM - (d+m+\gamma+bu_2)I,\\
\frac{dR}{dt} &= -(d+\alpha)R + (m\tau_1 + bu_2\tau_2)I,\\
\frac{dV}{dt} &= u_1 a - (d+\beta)V,\\
\frac{dL}{dt} &= rL - \sigma LI - \tau_3 u_3 L,\\
\frac{dM}{dt} &= \sigma LI - \tau_4 u_3 M - \delta M.
\end{aligned}
\tag{1.1}
$$

The initial condition of the system (1.1) are $S(0) > 0, I(0) > 0, R(0) > 0, V(0) \geqslant 0, L(0) \geqslant 0, M(0) \geqslant 0$.

The paper is organized in the following way: In Section 2, we describe the dynamical behavior of the system with fixed control. In particular we have studied about the boundedness of the system, possible equilibria and their stability and possible conditions under which the system will be disease free. Some simulations are also provided in this section to support our analytical results. in Section 3, we formulate an optimal control problem to minimize the infected individuals as well as the cost required for using the controls. Various simulation works on this optimal control problem are given in Section 4. Finally, in Section 5, we have provided the concluding remarks based on our analysis in the earlier sections.