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FPGA realization of multi-scroll chaotic oscillators

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Chaotic oscillators have been realized using field-programmable gate arrays (FPGAs) showing good results. However, only 2-scrolls have been observed experimentally, and all reported works use commercially-available software tools for FPGA synthesis. In this manner, as a first contribution we show the FPGA realization of two multi-scroll chaotic oscillators that are characterized by their maximum Lyapunov exponent (MLE) for generating from 2- to 6-scrolls. The first multi-scroll chaotic oscillator is based on saturated function series and the second on Chua's circuit. As a second contribution, we show their hardware realization by applying two numerical methods: Forward Euler (FE) and Runge Kutta (RK). The advantage of realizing those multi-scroll chaotic oscillators is that one can avoid the use of multiplier entities, thus optimizing FPGA resources and increasing the processing speed, as we show by realizing single constant multiplication (SCM) blocks. The experiments are verified by performing co-simulation for an FPGA Spartan 3 of Xilinx. Finally, experimental results are shown for different values of MLE (already optimized) for both multi-scroll chaotic oscillators, and the FPGA used resources are listed for generating 6-scrolls when applying FE and RK.

1. Introduction

Chaotic oscillators have been investigated to guarantee chaotic regime by optimizing the maximum Lyapunov exponent (MLE) and ensuring a good distribution of trajectories in the phase-space portraits for generating multi-scroll attractors [1]. Those optimized chaotic oscillators have been realized with different kinds of electronic devices, trying to generate the higher number of scrolls [2]. However, it has been observed the difficulty of programming different values for the parameters involved in the mathematical model of a chaotic oscillator. For example: in [1] the MLE of two multi-scroll chaotic oscillators is optimized; one based on saturated function series and the other on Chua's circuit. Both chaotic oscillators can have different values of their coefficients as well as their piecewise-linear (PWL) function can change, so that one needs to use precision circuit elements to tune the desired value for the coefficient or PWL function, but the major problem is when tuning values having more than three decimals, i.e. <0.001. Because precision resistors are used to tune those decimal values, and since they have very high variation, then we appeal using field-programmable gate arrays (FPGAs) to program coefficient parameters and PWL functions having three or more decimals.

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FPGAs have been used to realize applications on chaotic systems. For example: in [3] a chaotic map with high MLE is implemented into an FPGA, in [4] an FPGA-based 3D chaotic system is realized by an auto-switched numerical resolution of multiple three dimensional continuous chaotic systems, in [5] an FPGA-realization of a self-synchronizing chaotic decoder in the presence of noise is presented, in [6] the FPGA design for a pseudo-random number generator is given, in [7] other FPGA design for a discrete chaotic map is presented, and more recently: in [8] an implementation of an FPGA-based real time novel chaotic oscillator is introduced. Although the MLE and phase portraits are shown, they are not optimized; only 2-scrolls are generated, and the hardware architecture from VHDL (Very High Speed Integrated Circuit Hardware Description Language) simulations is not described. In this manner, this article shows the FPGA realization of two multi-scroll chaotic oscillators having optimal and different MLEs: one based on saturated function series and the other based on Chua's circuit. In addition, their FPGA realization is described by applying two numerical methods (Forward Euler and Runge Kutta) for solving their models given by three first-order differential equations. We show their hardware description, the computer arithmetic, and their co-simulation for a low-cost FPGA from Xilinx. Both chaotic oscillators are realized for generating from 2- to 6-scrolls. At the end, we show several experimental results for different MLEs, and the FPGA used resources are listed for generating 6-scrolls when applying both numerical methods.

2. Multi-scroll chaotic oscillators

In [1], two multi-scroll chaotic oscillators are described: one based on saturated function series and the second on Chua's circuit. They are optimized by maximizing their MLE and minimizing the dispersions of their trajectories in the phase portraits among all scrolls in an attractor.

2.1. Chaotic oscillator based on saturated function series

This multi-scroll chaotic oscillator is described by three differential equations, as given by (1), where $a,\ b,\ c$, and d_1 are positive constants that can have values in the interval $[0,1]$. The dynamical system is controlled by a PWL approximation that describes the saturated function series f, which is obtained as follows: Let f_0 be the saturated function described by (2), where: $1/m$ is the slope of the middle segment and $m>0$; the upper radial $\{f_0(x;m)=1\ |x>m\}$, and the lower radial $\{f_0(x;m)=-1\ |x<-m\}$ are called *saturated plateaus*, and the segment $\{f_0(x;m)=x/m \mid |x|\leqslant m\}$ between the two saturated plateaus is the *saturated slope*.

$$\begin{aligned}\dot{x}&=y,\\ \dot{y}&=z,\\ \dot{z}&=-ax-by-cz+d_1f(x;m),\end{aligned}\tag{1}$$

$$f_0(x;m)=\begin{cases}1, & \text{if } x>m,\\ \frac{x}{m}, & \text{if } |x|\leqslant m,\\ -1, & \text{if } x<-m,\end{cases}\tag{2}$$

The saturated functions described by f_h and f_{-h} can be defined as:

$$f_h(x;m,h)=\begin{cases}2, & \text{if } x>h+m,\\ \frac{x-h}{m}, & \text{if } |x-h|\leqslant m,\\ 0, & \text{if } x<h-m\end{cases}\tag{3}$$

and

$$f_{-h}(x;m,-h)=\begin{cases}0, & \text{if } x>h+m,\\ \frac{x-h}{m}, & \text{if } |x-h|\leqslant m,\\ -2, & \text{if } x<h-m,\end{cases}\tag{4}$$

where h is the *saturated delay*, and $h>m$. Therefore, the saturated function series for generating s scrolls can be defined by (5) [1], for $s>2$.

$$f(x;m)=\sum_{i=0}^{s-2}f_{2i-s+2}(x;m,2i-s+2).\tag{5}$$

2.2. Chaotic oscillator based on Chua's diode

The multi-scroll chaotic oscillator based on Chua's circuit is described by (6) [1], where α and β are positive and real constants, and $g(x)$ is the PWL function known as Chua's diode and defined by (7), where m_i are negative slopes. The constants b_i